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ISOQUANT SFB1225

Towards in-medium $Q\bar{Q}$ phenomenology from lattice QCD spectral functions

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Institute for Theoretical Physics
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References:

With Y. Burnier PRL 111 (2013) 182003 and PLB753 (2016) 232
With Y. Burnier and O. Kaczmarek PRL 114 (2015) 082001,
JHEP 1512 (2015) 101; JHEP 1610 (2016) 032
With S. Kim and P. Petreczky PRD91 (2015) 054511
With J. Pawlowski arxiv:1610.09531

Physics Motivation: Quarkonium

- A hard probe in heavy-ion collisions: early production, samples full QGP evolution

Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q >> T_{med}$

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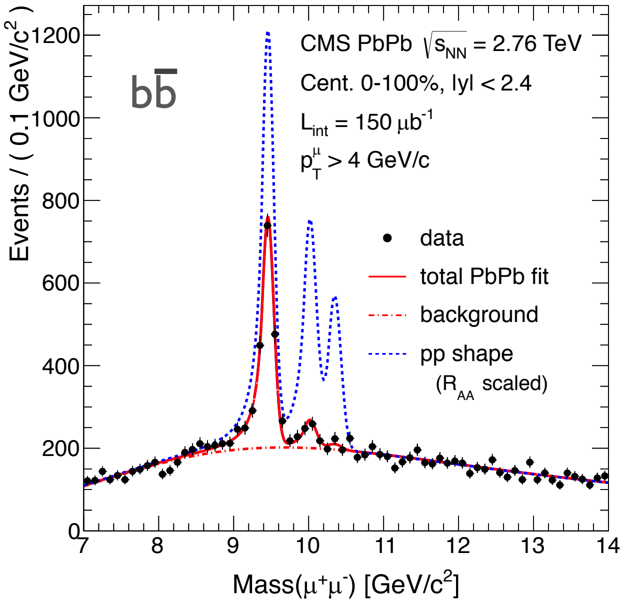

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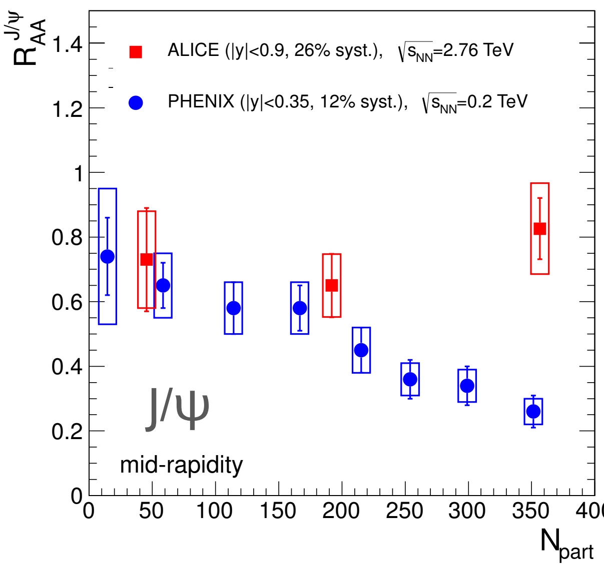

ALICE, PRL 109, 072301 (2012)
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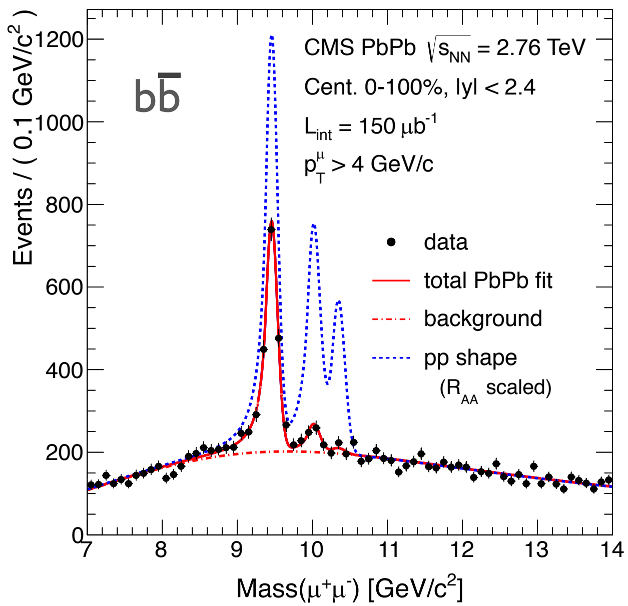

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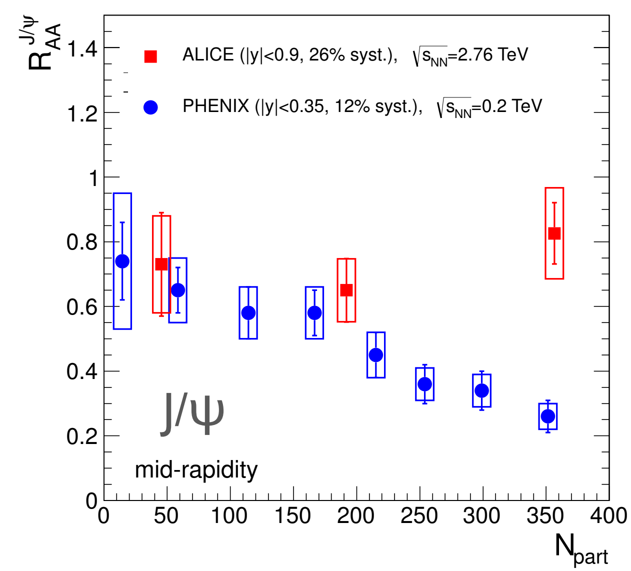

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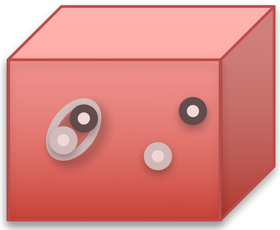
- Theory goal: 1st principles insight into in-medium $Q\bar{Q}$ in heavy-ion collisions

A two-pronged approach to $Q\bar{Q}$



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&
Static medium from lattice QCD

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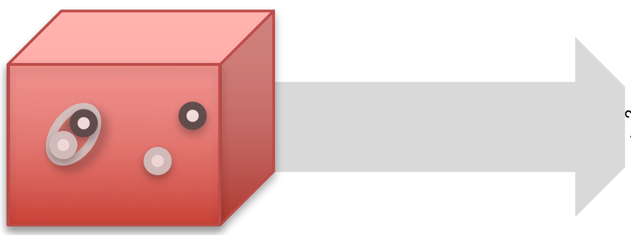

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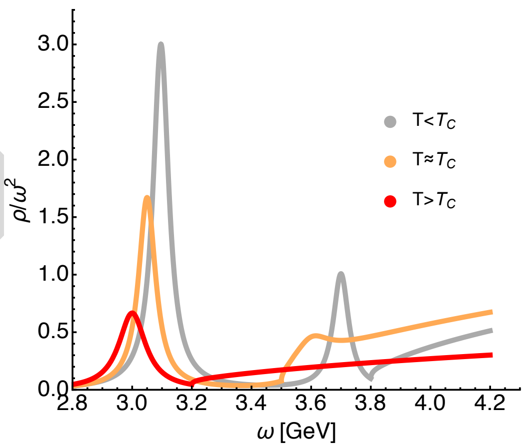
In-medium meson spectra



schematic depiction

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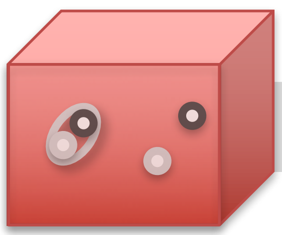

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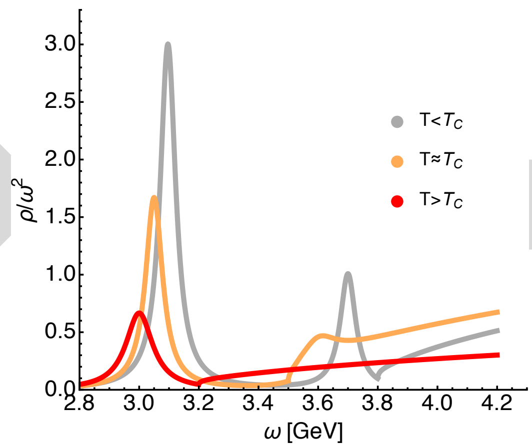

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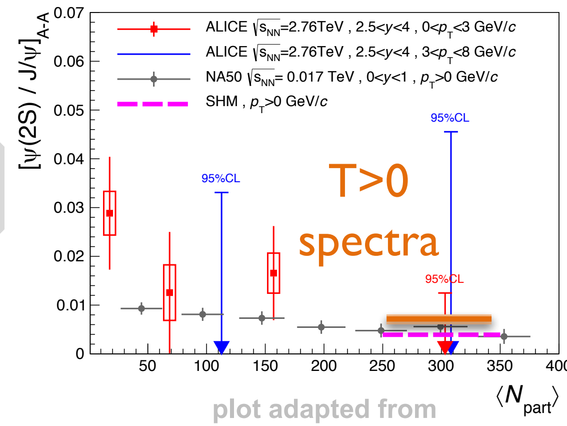
Observables
e.g. ψ' / J/ψ ratio



plot adapted from
ALICE Collaboration JHEP05(2016)179

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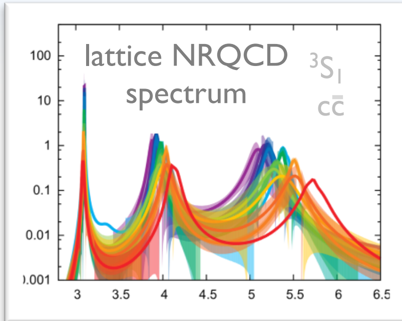
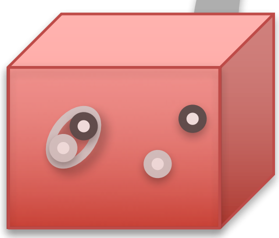


I. Direct reconstruction of lattice meson spectra in NRQCD
(limited resolution)



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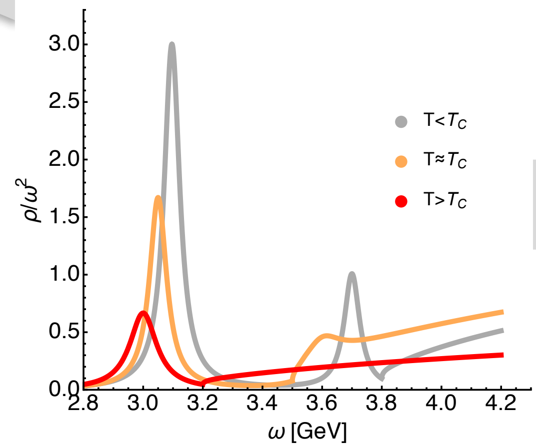

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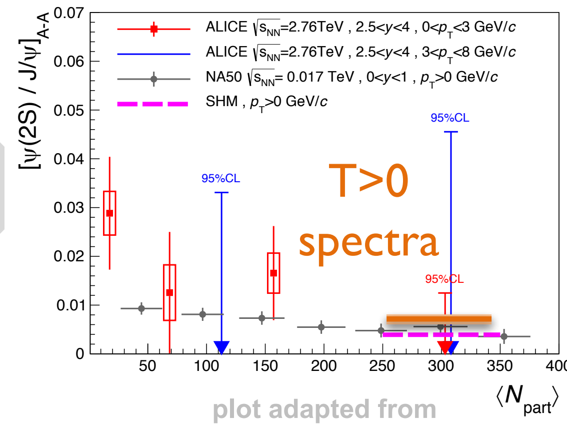

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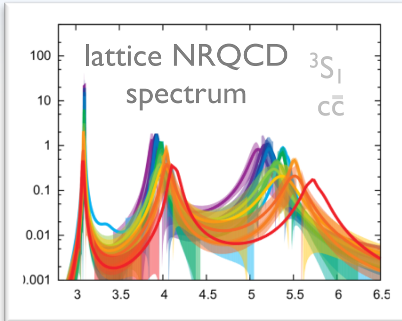

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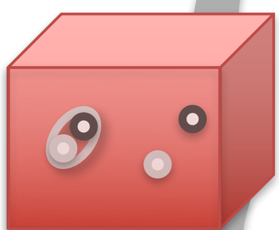

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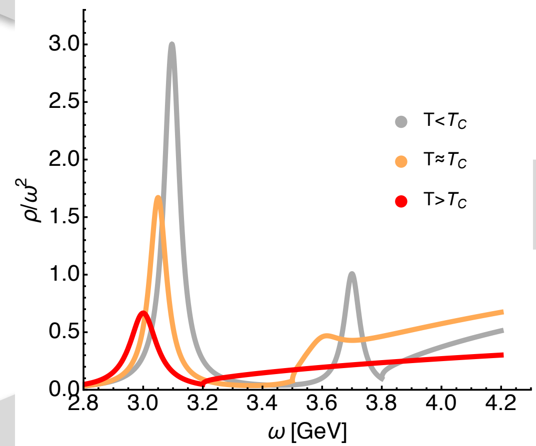

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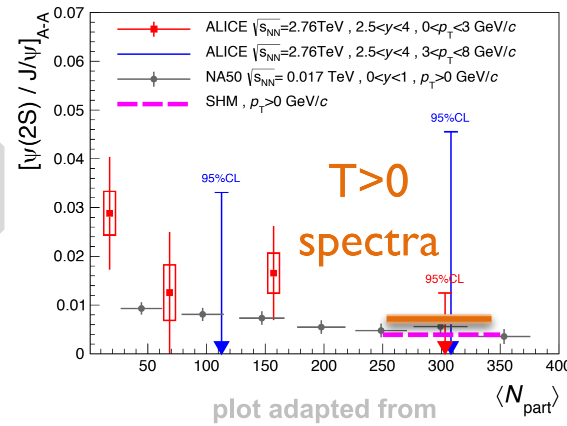

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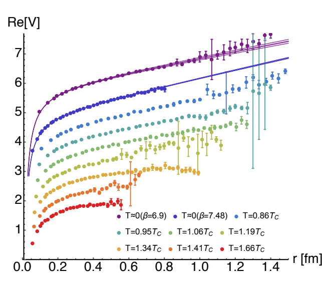

II. Via QQ potential from the lattice QCD Wilson loop
(currently static potential only)

A common challenge

- Dynamical information e.g. spectral functions not directly accessible on the lattice

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- Bayesian continuum limit $N_\tau \Rightarrow \infty,\ \Delta D/D \rightarrow 0$ exponentially hard to reach

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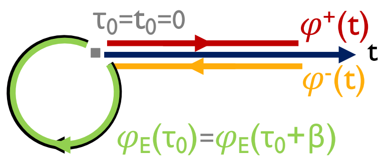

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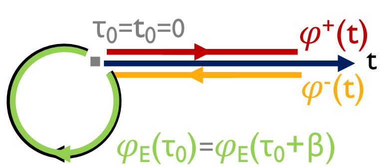

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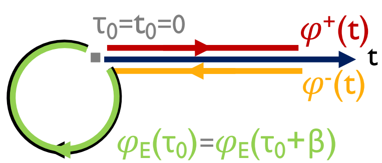

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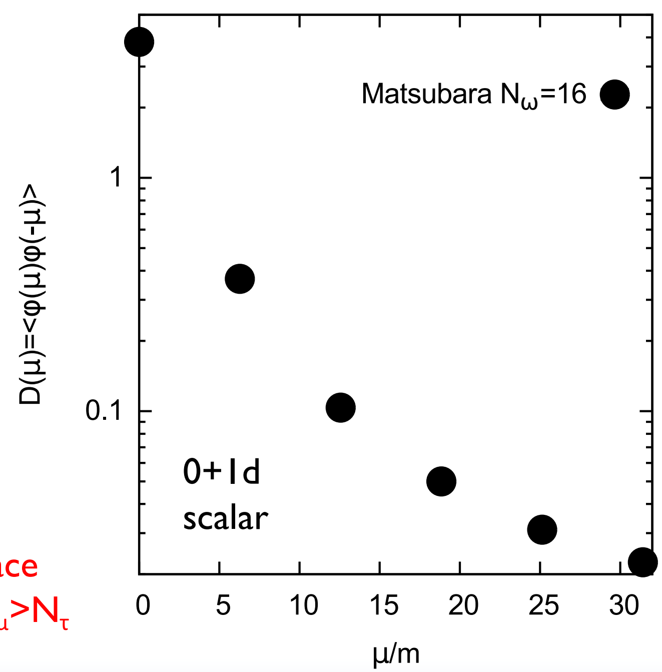

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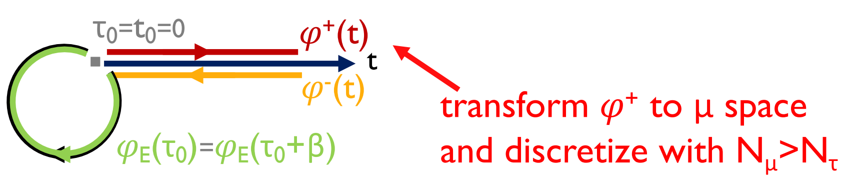
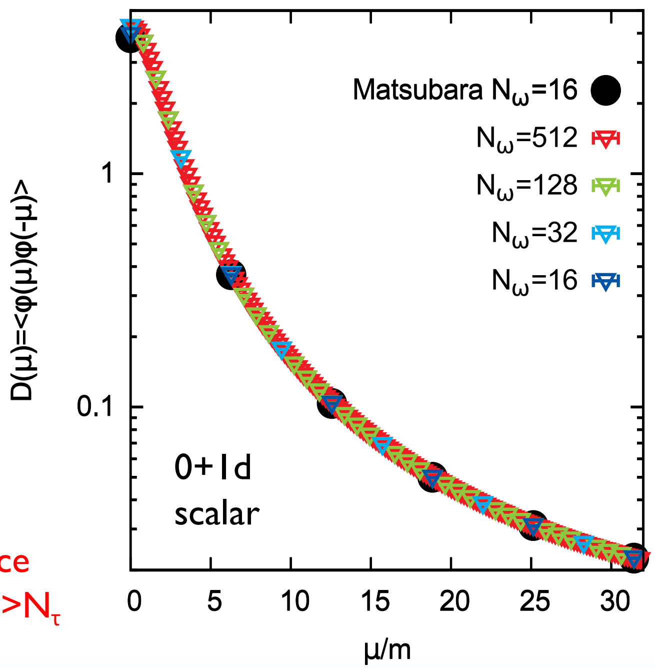
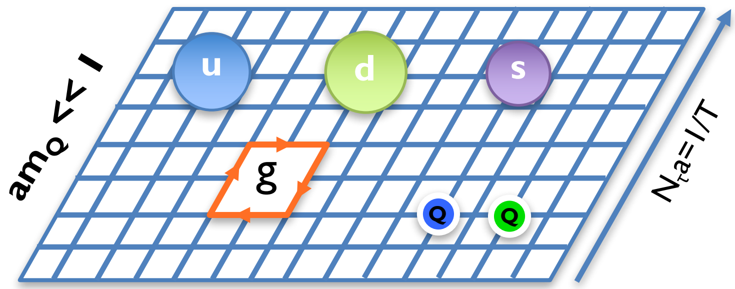




In-medium quarkonium spectral functions from lattice QCD

I. Direct determination: NRQCD

Relativistic treatment of light and heavy d.o.f.



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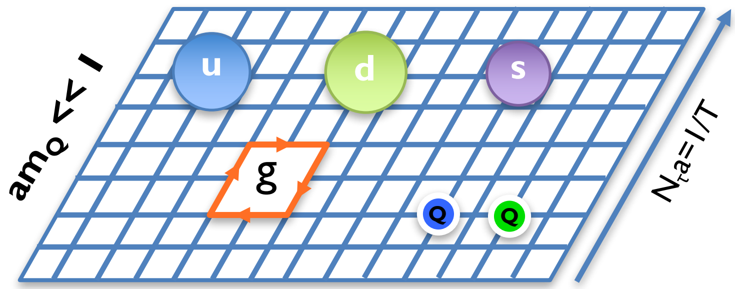

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$$\frac{\Lambda_{QCD}}{m_Q} \ll 1$$

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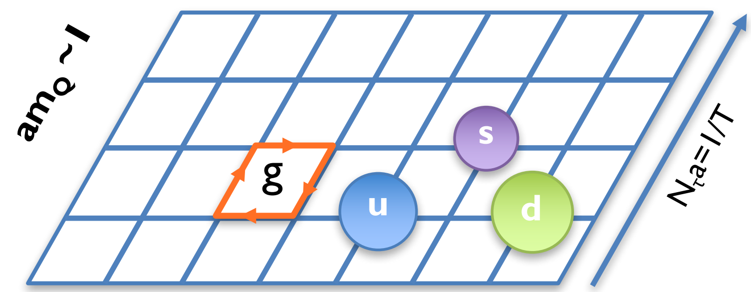

Lattice QCD simulation without QQ̄

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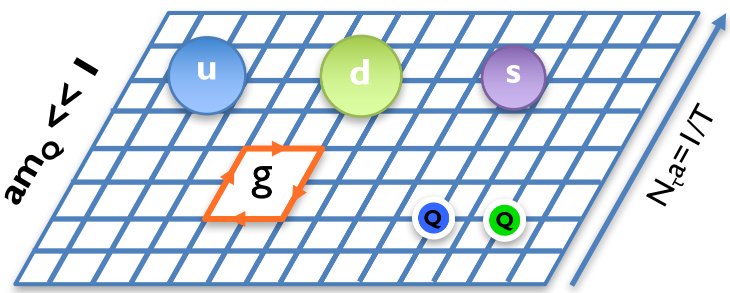
Kin. equil. non-relativistic $Q\bar{Q}$ in a background of light medium d.o.f.



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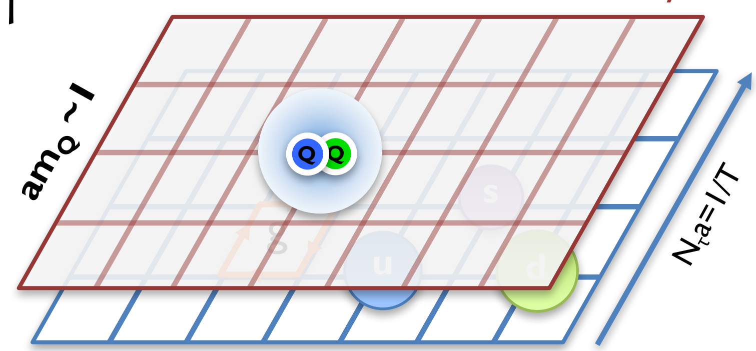
$Q\bar{Q}$ in NRQCD effective theory



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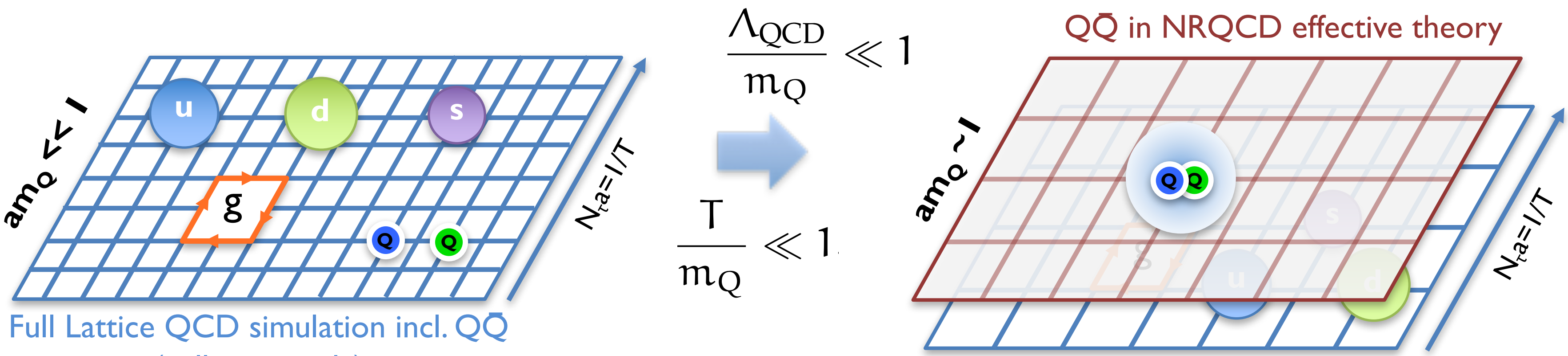

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Lattice QCD simulation without $Q\bar{Q}$

- Lattice Non-Relativistic QCD (NRQCD) well established at T=0, applicable at T>0
  - no modeling, systematic expansion of QCD action in $1/m_Q a$, includes v≠0 contributions

Thacker, Lepage Phys.Rev. D43 (1991) 196-208

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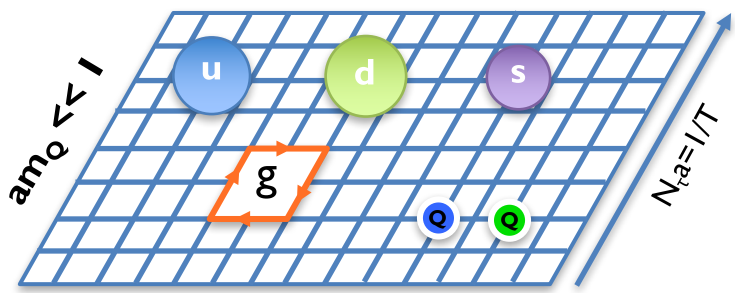


Full Lattice QCD simulation incl. $Q\bar{Q}$ (still too costly)

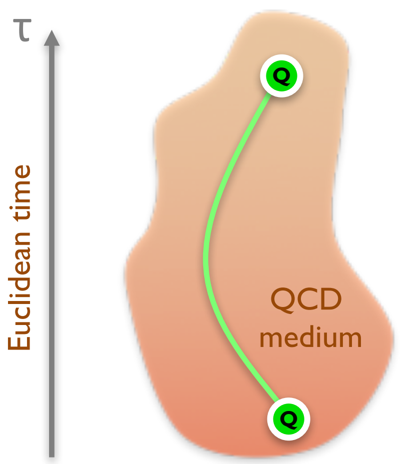
$$\frac{\Lambda_{QCD}}{m_Q} \ll 1$$

$$\frac{T}{m_Q} \ll 1$$

Lattice QCD simulation without $Q\bar{Q}$

- Lattice Non-Relativistic QCD (NRQCD) well established at T=0, applicable at T>0
  - no modeling, systematic expansion of QCD action in $1/m_Q a$, includes v≠0 contributions
    Thacker, Lepage Phys.Rev. D43 (1991) 196-208
- State-of-the-art: realistic simulations of the QCD medium by the HotQCD collab.
  HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - 48^3x12 N_f=2+1 HISQ action m_π=161MeV T= [140 – 407] MeV $m_b a$= [2.759 – 0.954]

Correlation functions in NRQCD



Non-rel. propagator of
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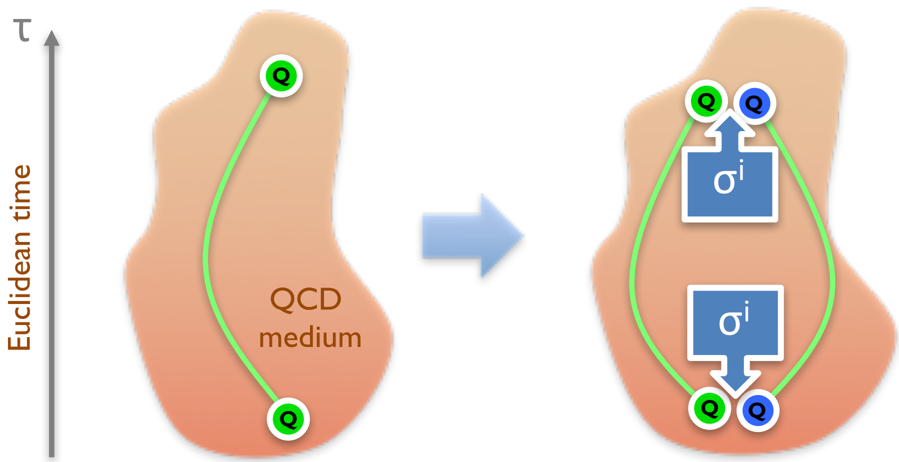

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Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

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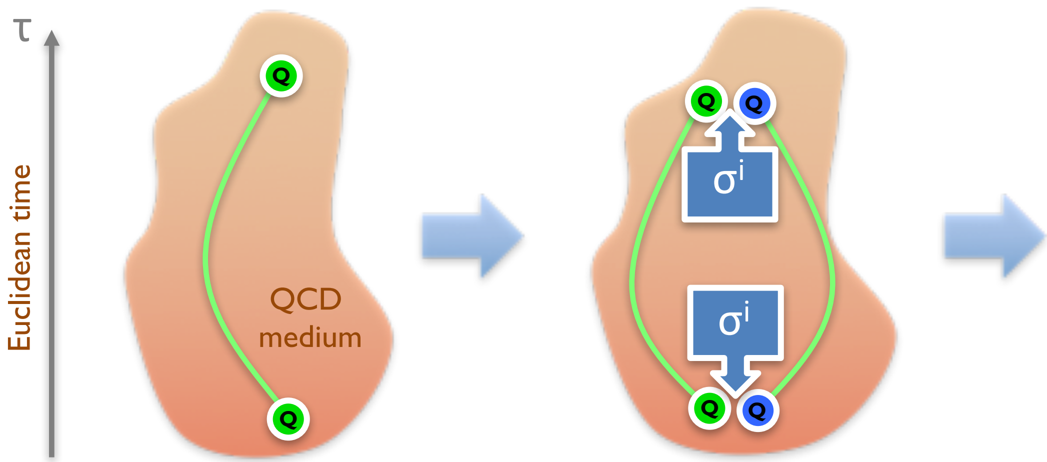

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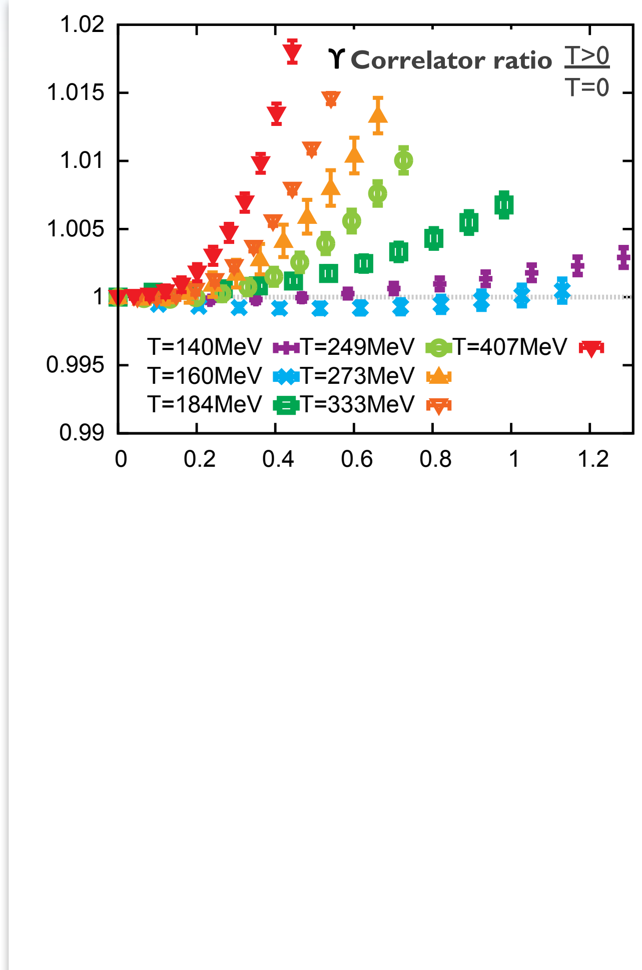

S.Kim, P.Petreczky, A.R. PRD91 (2015) 054511 and in preparation

Ratio of T>0 and T≈0 correlators: estimate of overall in-medium effects

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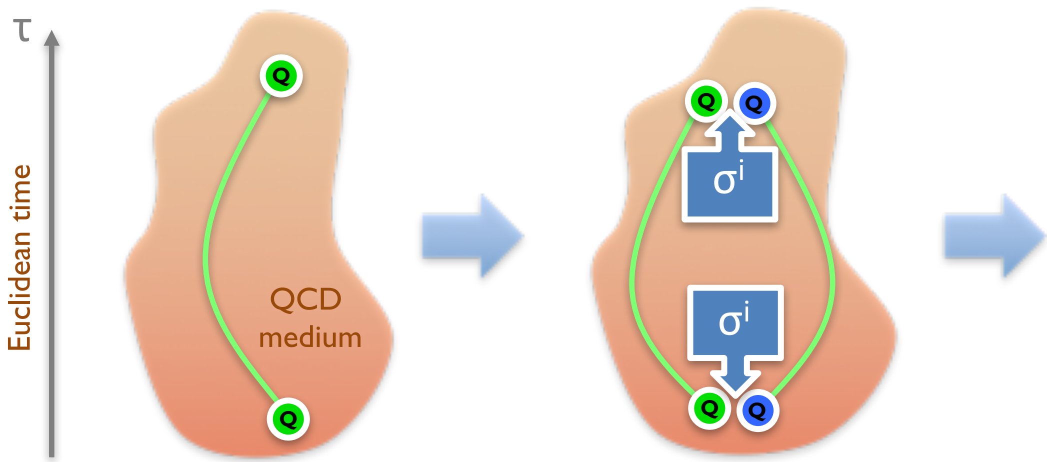

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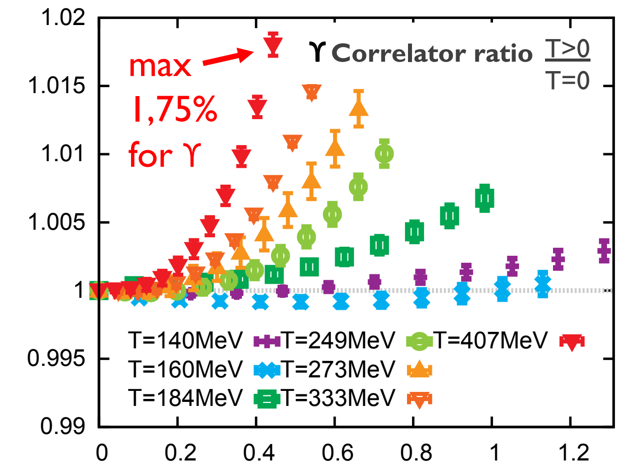

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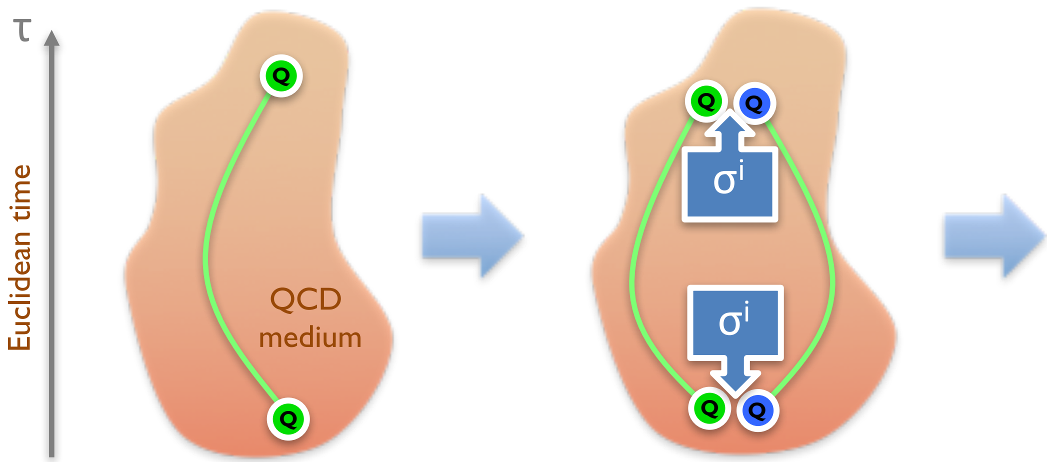

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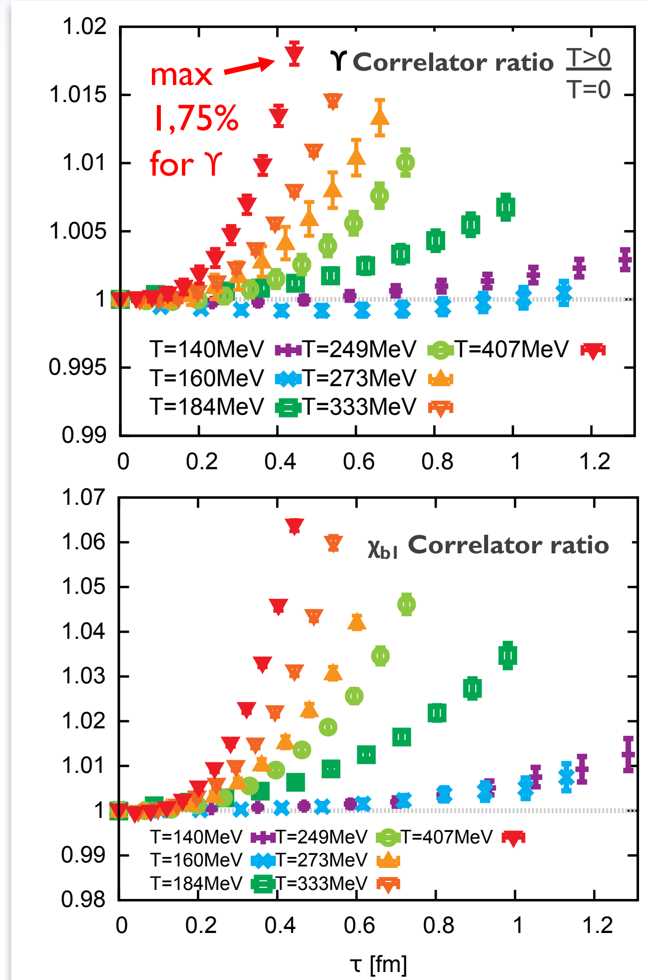

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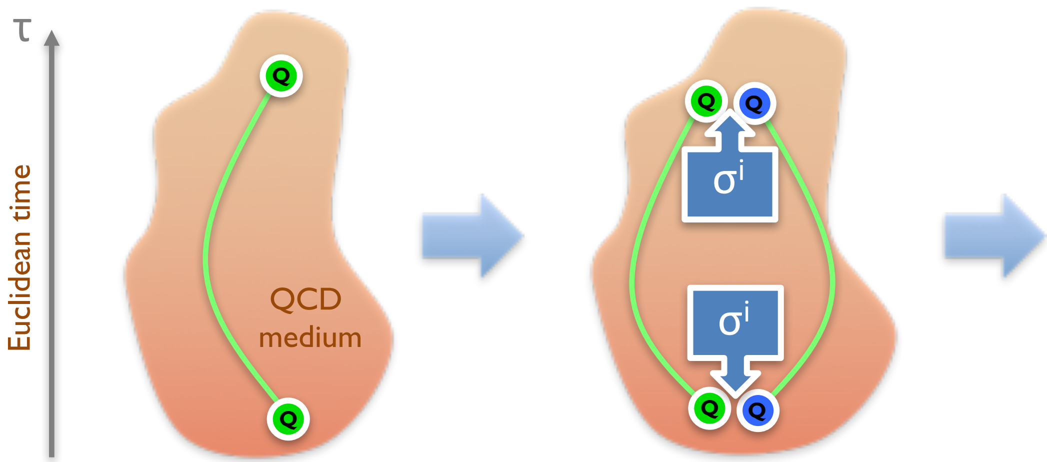

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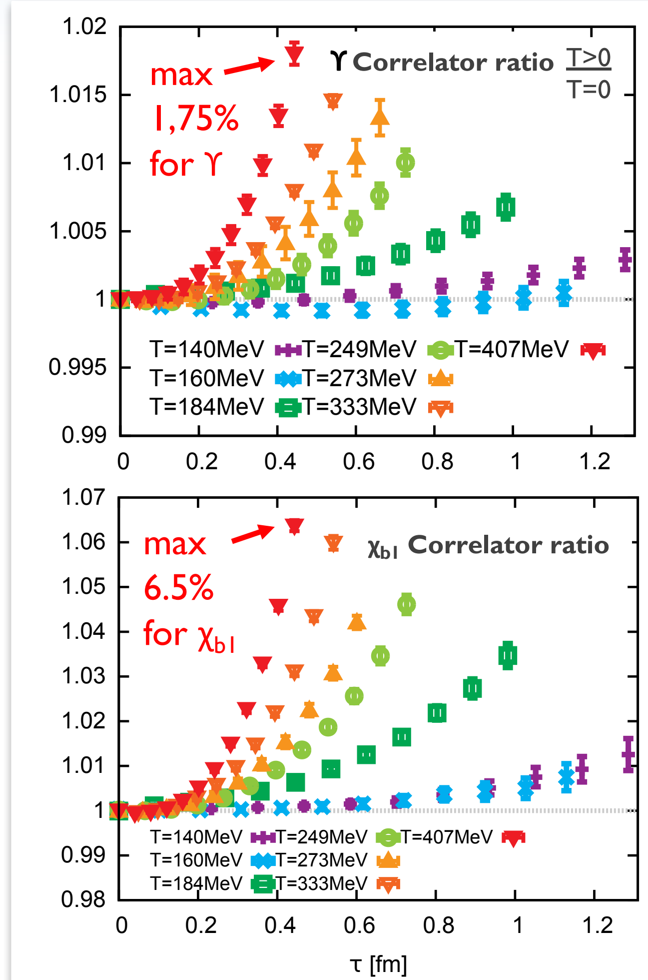

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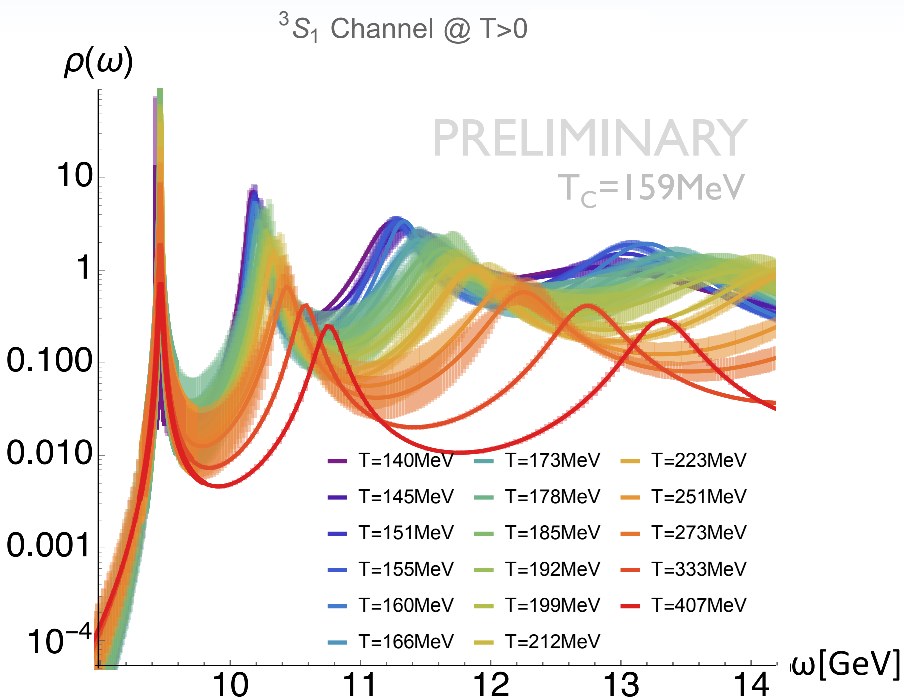
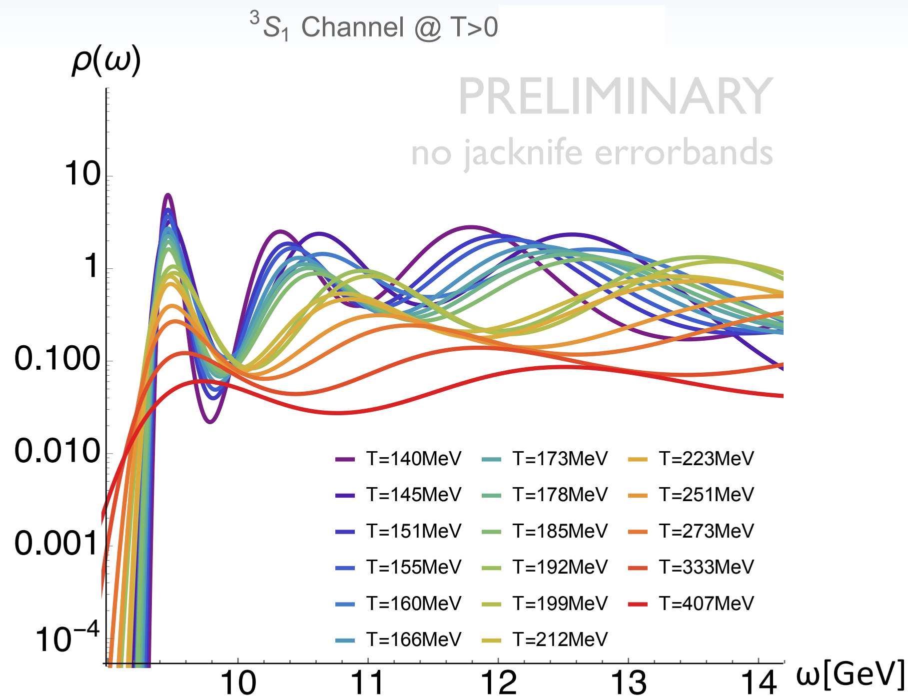
Bottomonium NRQCD S-wave spectra

S.Kim, P.Petreczky, A.R. in preparation





For a prior MEM based investigation see
G.Aarts et.al. [FASTSUM] JHEP 1407 (2014) 097

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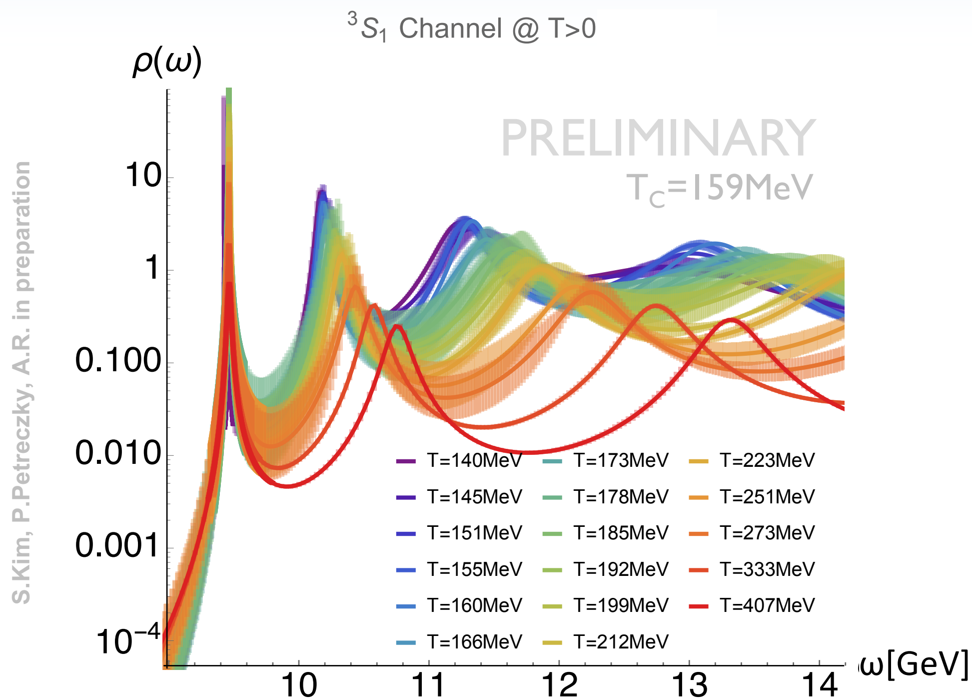

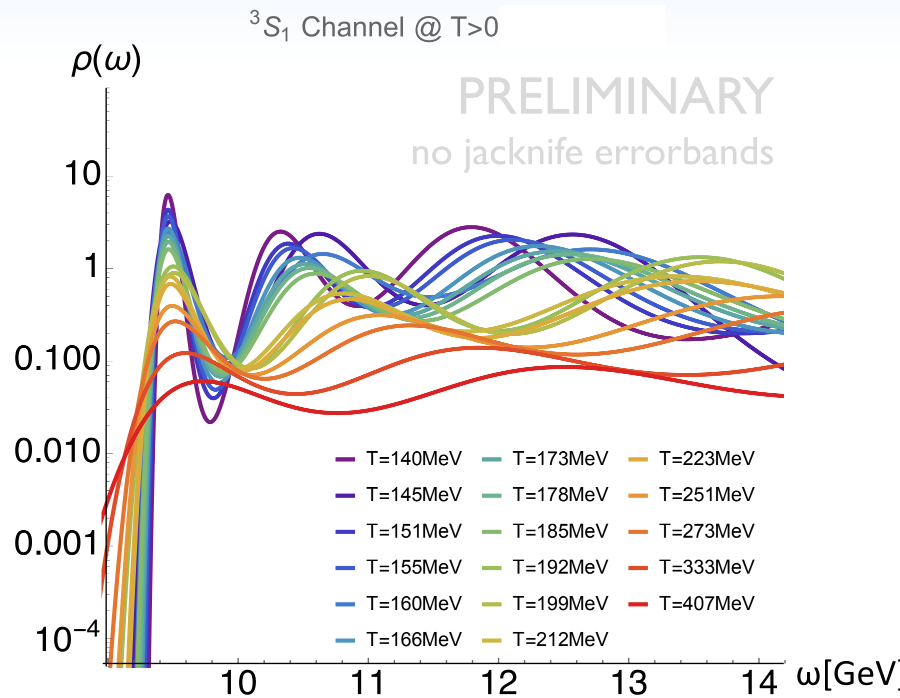

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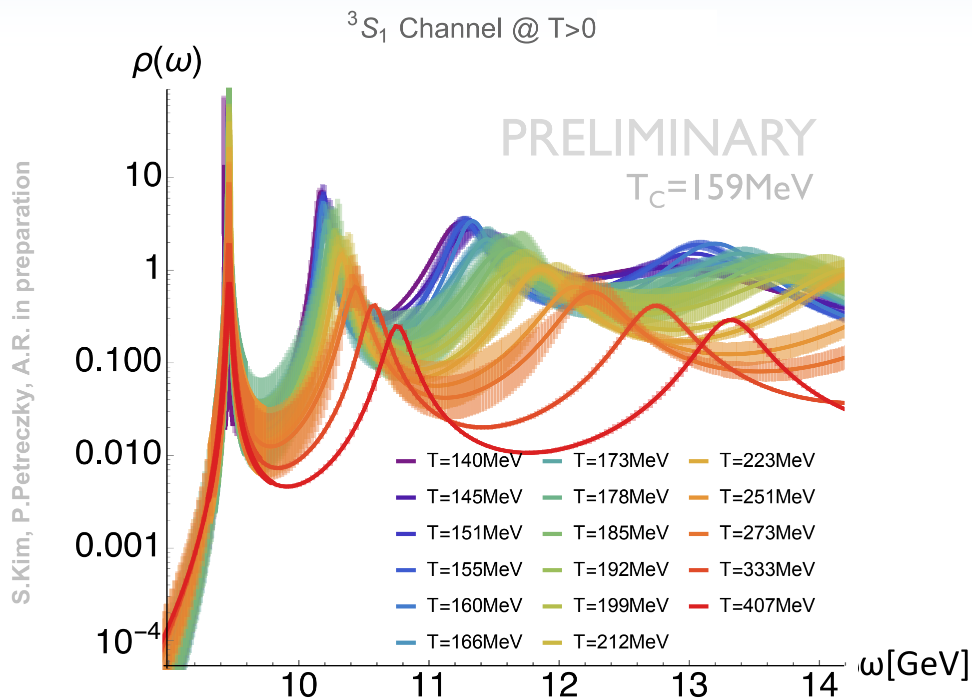

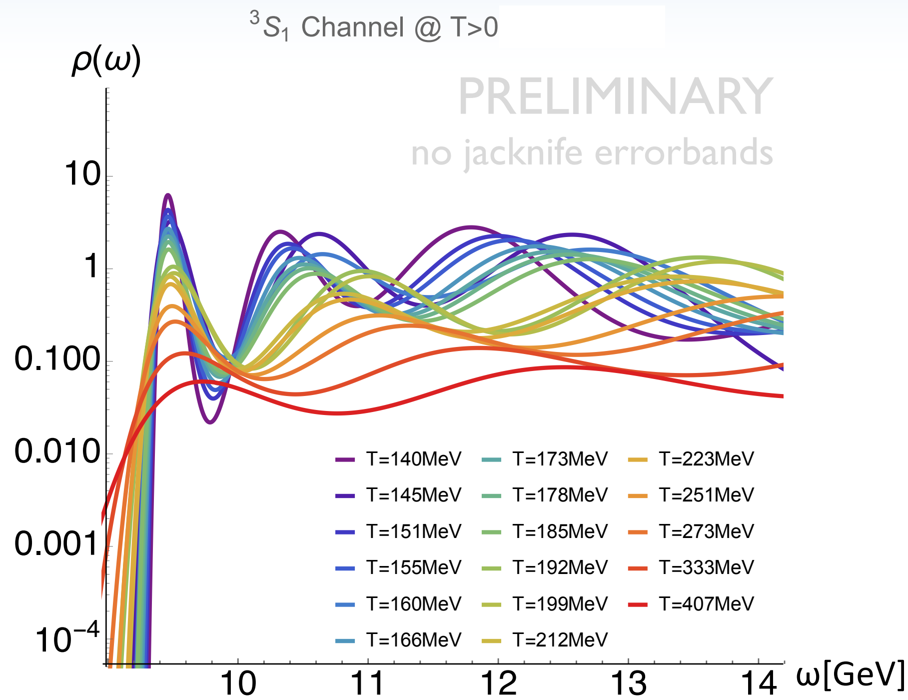

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- MEM: around T=333 MeV only washed out bump visible

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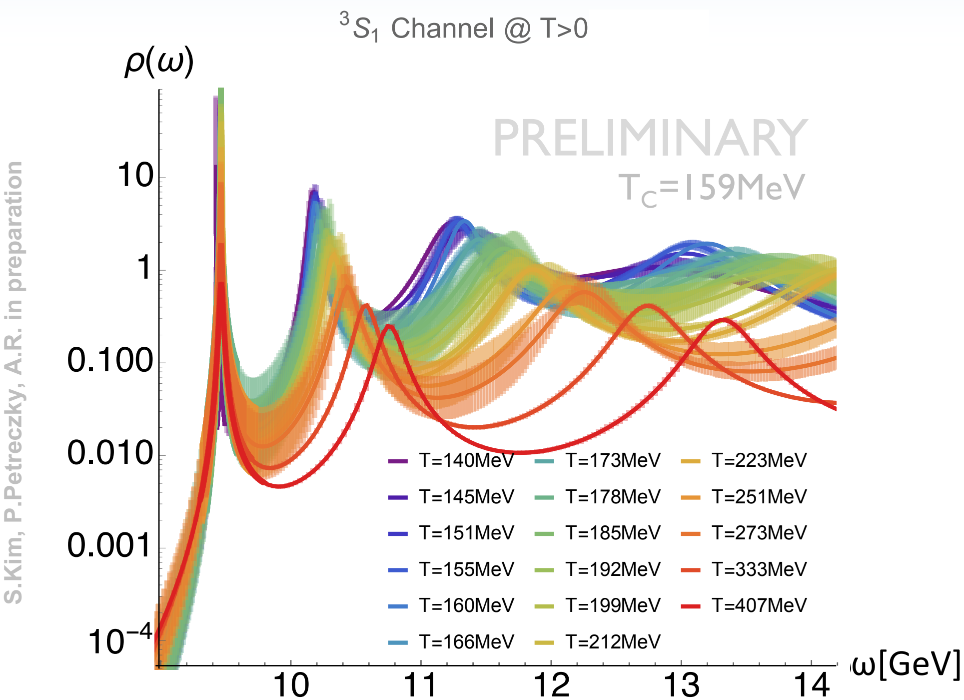

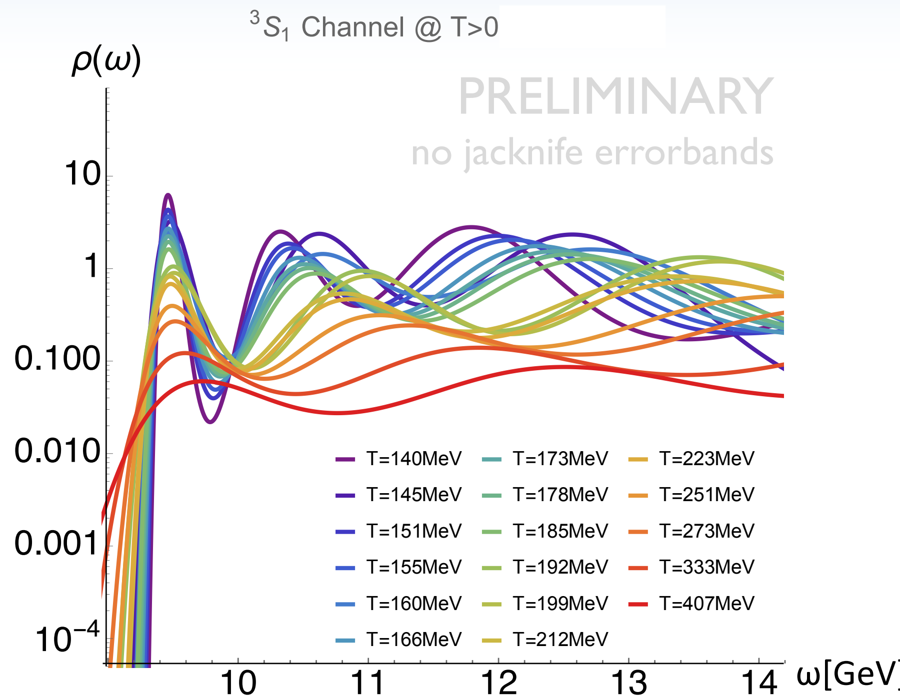

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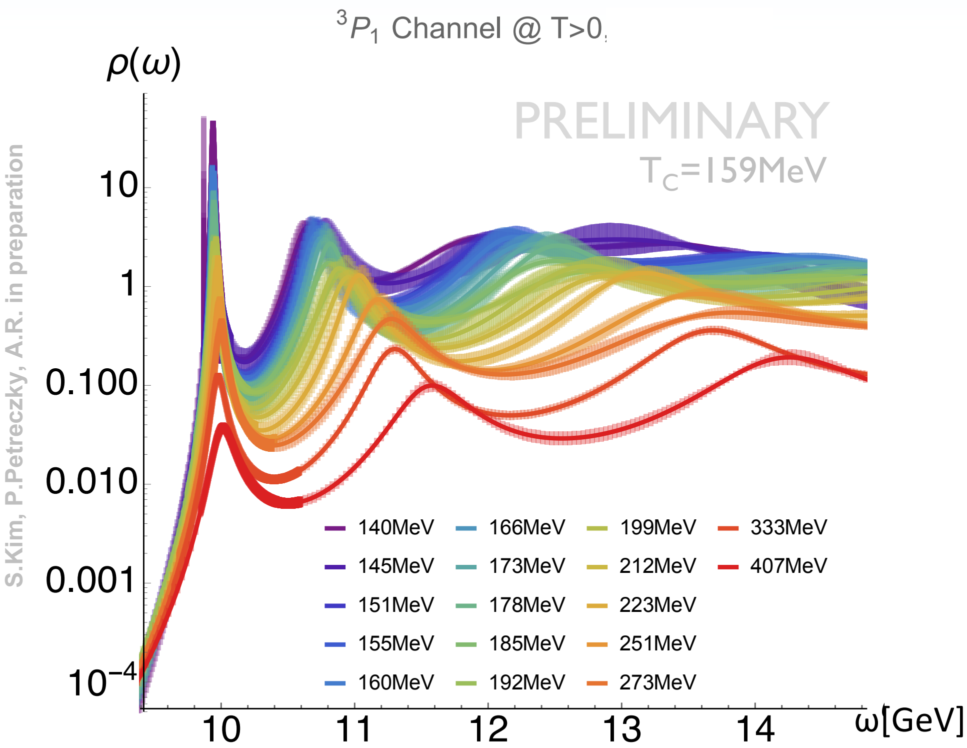
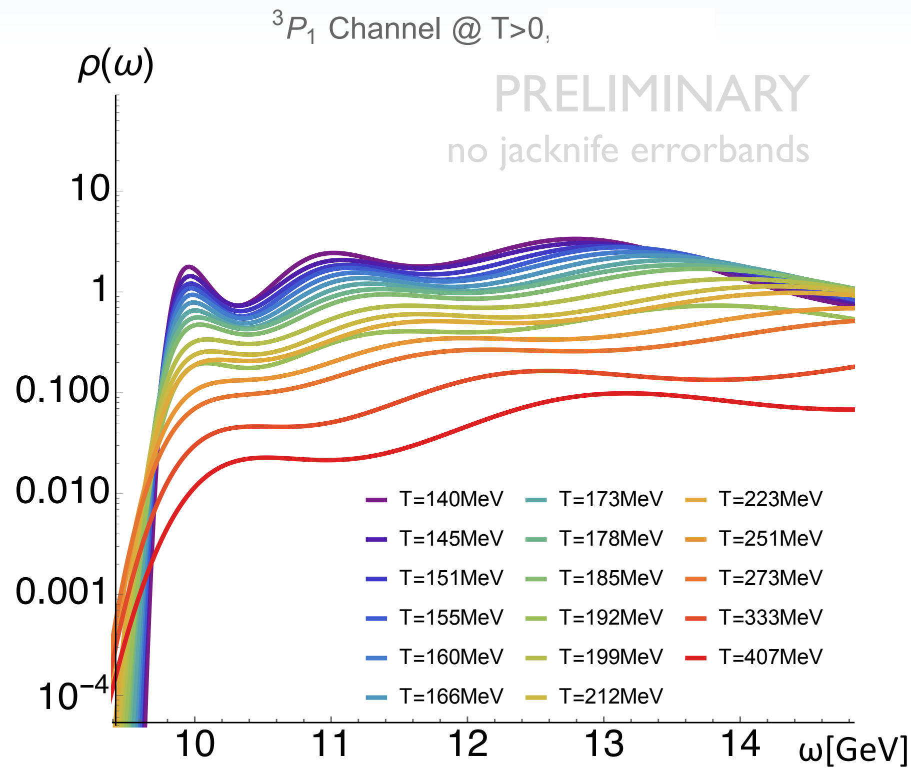
- Systematics: MEM over smoothing, BR ringing – use both methods to bracket

Bottomonium NRQCD P-wave spectra





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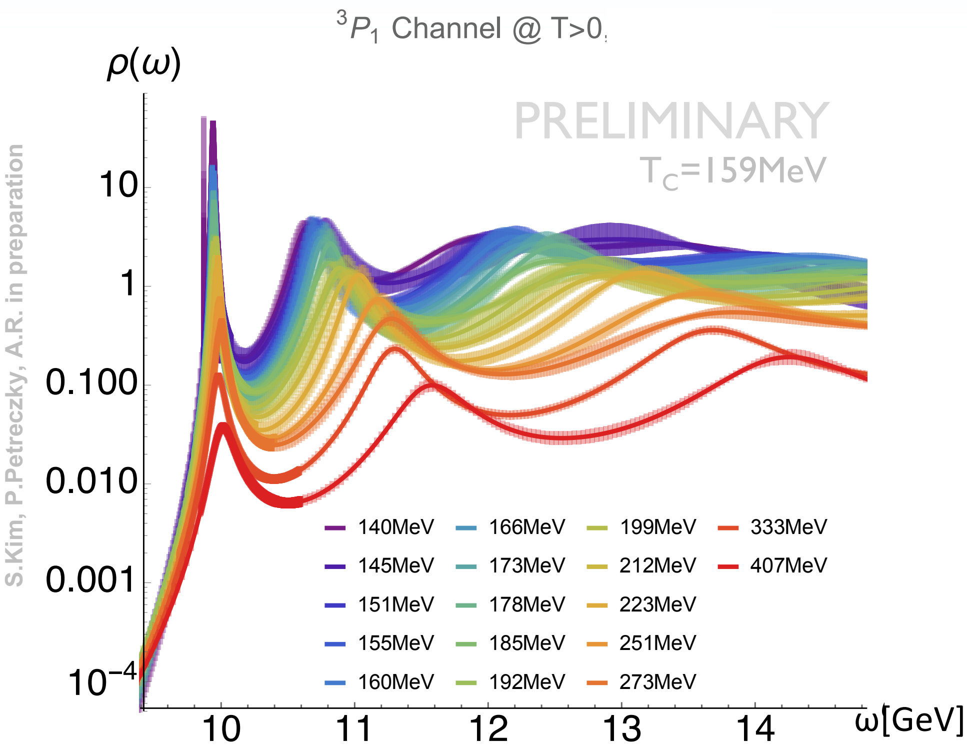

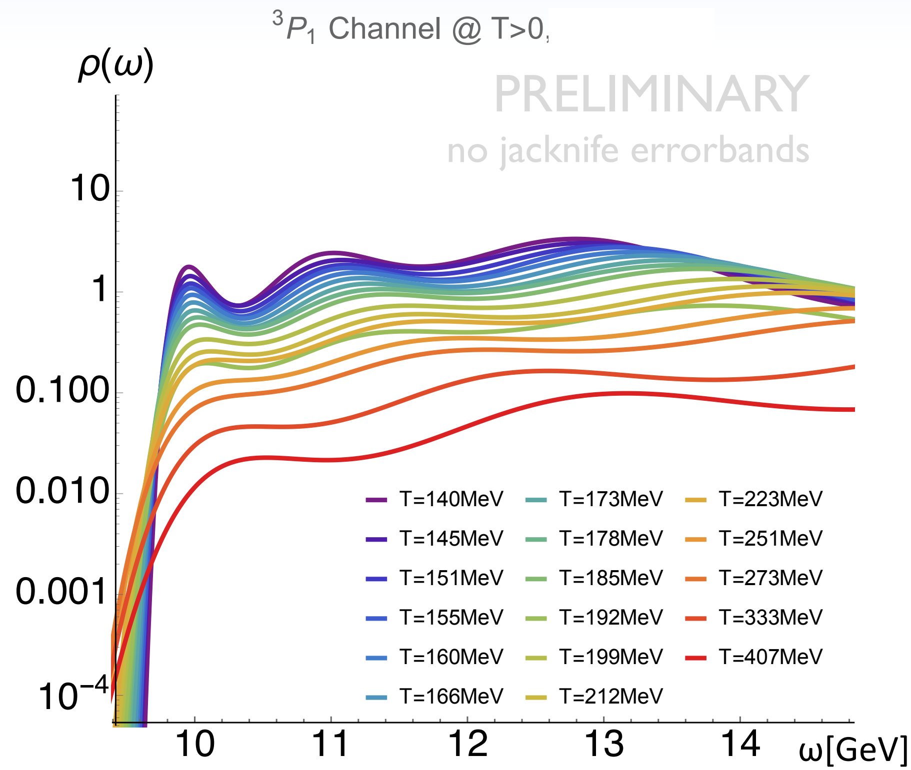

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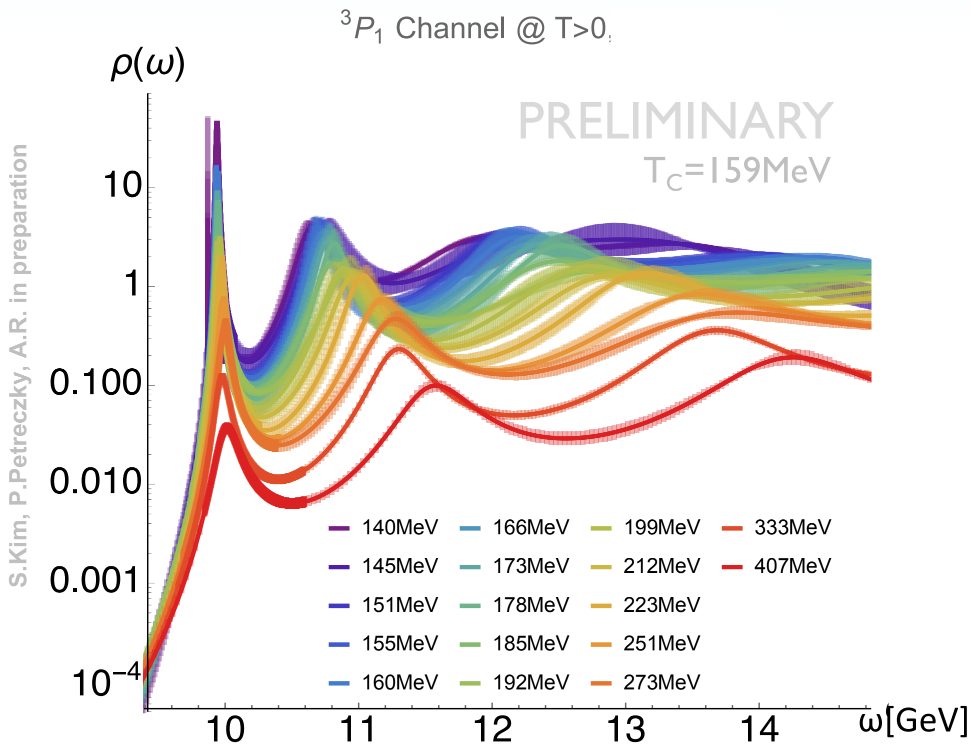

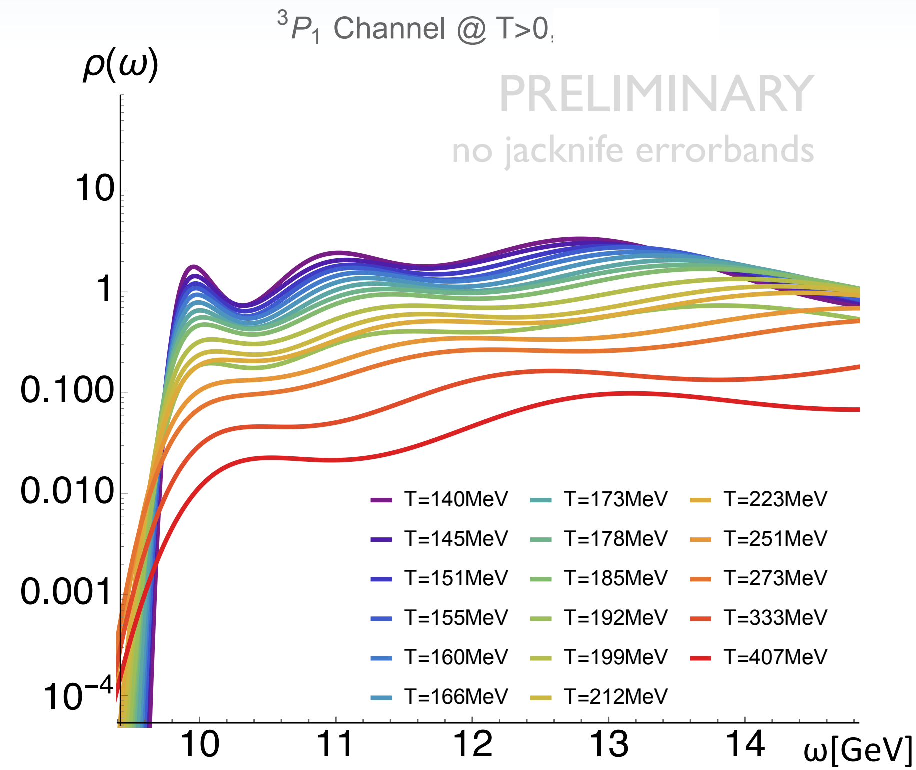

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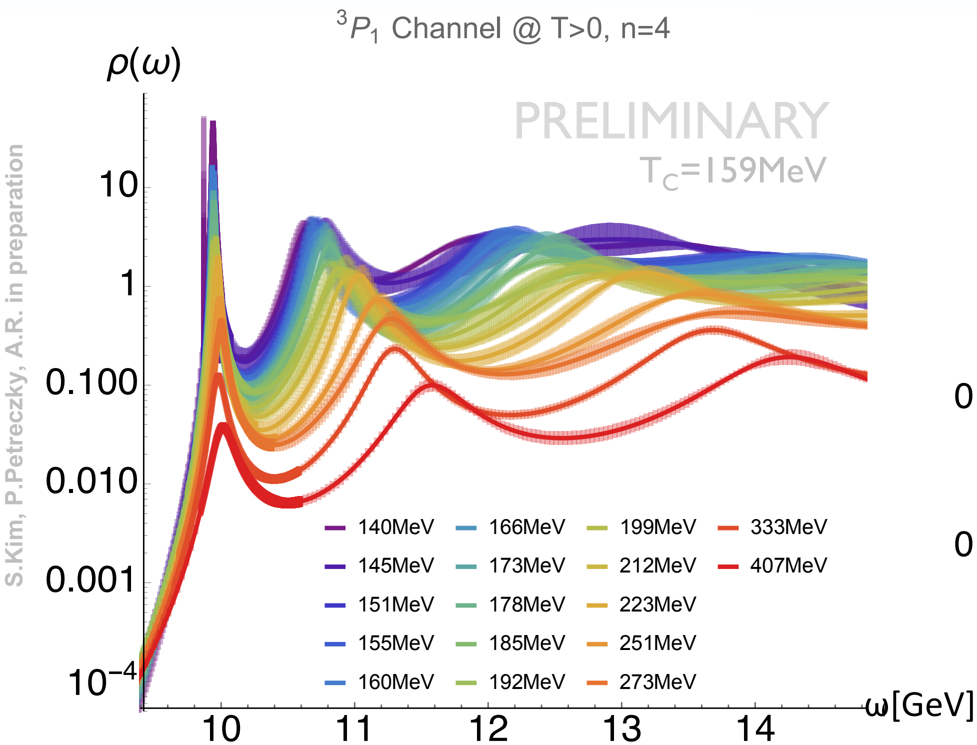
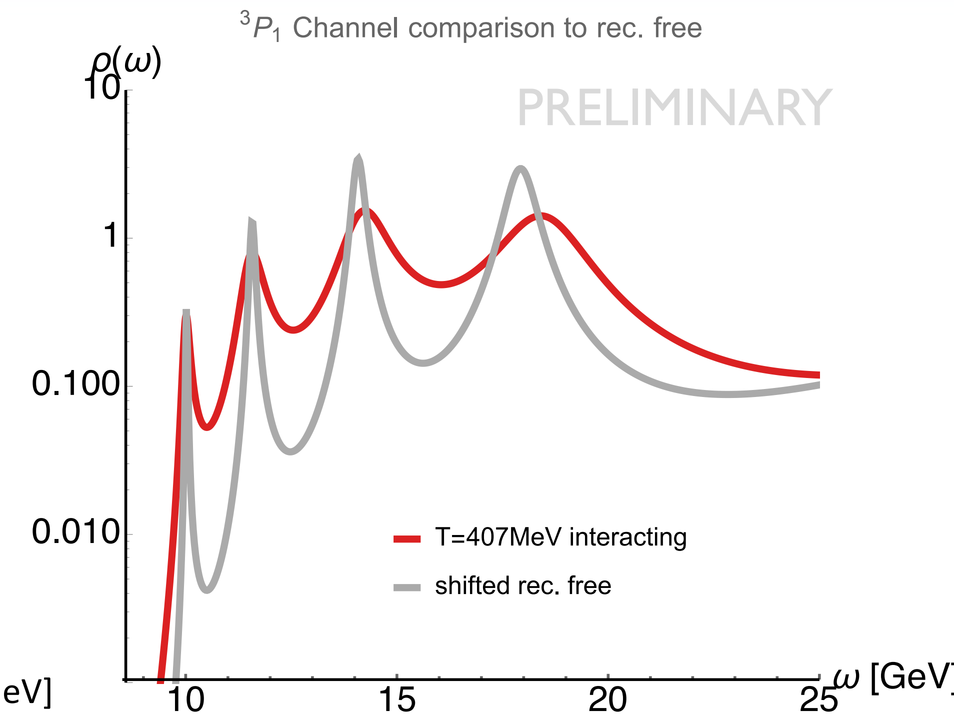
- BR: Can we disentangle numerical ringing from actual physics at T=407MeV

How to identify BR method artifacts?





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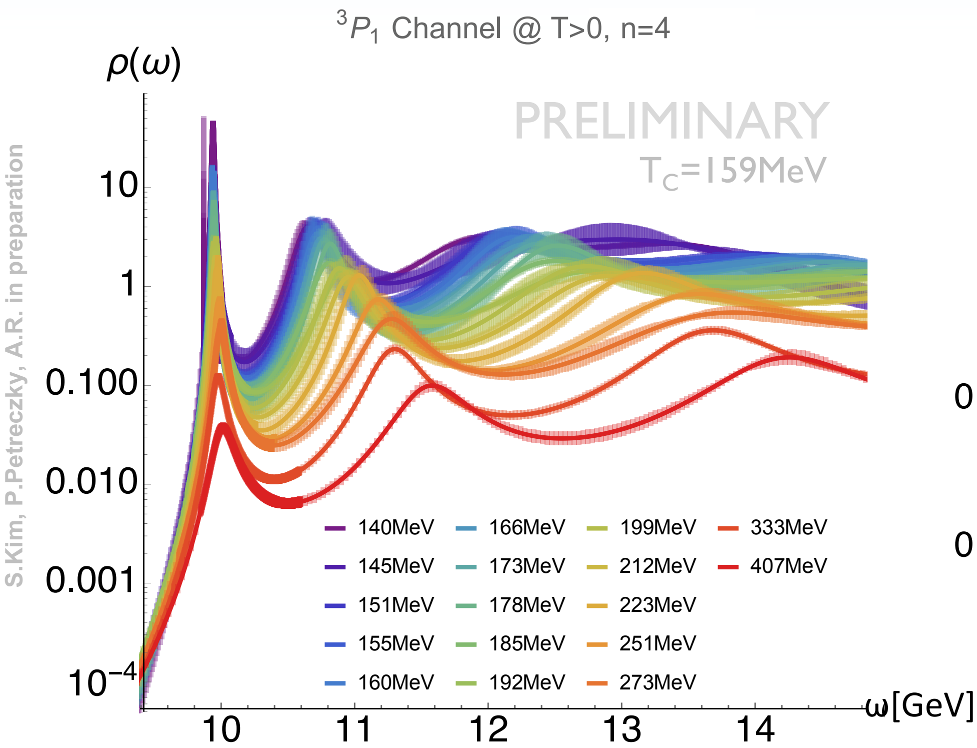

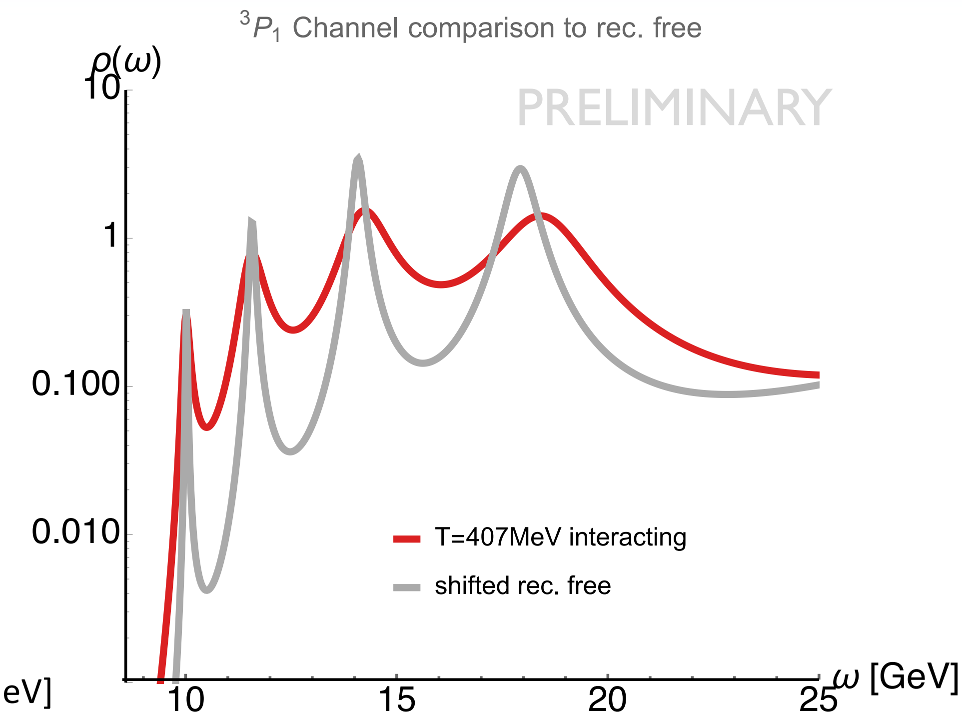

- Distinguish ringing and actual bound state signal via free spectral functions
- Hints at disappearance of genuine bound state feature above T=333MeV
- Better understanding of Bayesian systematics (MEM,BR) but remains a challenge:
  increasing statistics S.Kim, P.Petreczky, A.R. In preparation or increasing N_τ G. Aarts et.al [FASTSUM] in preparation

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Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

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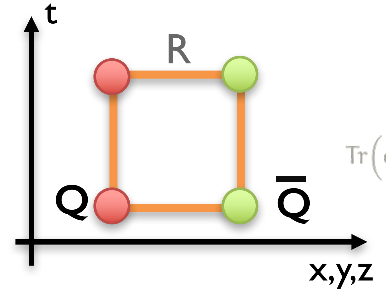

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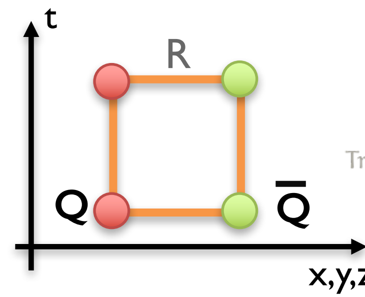

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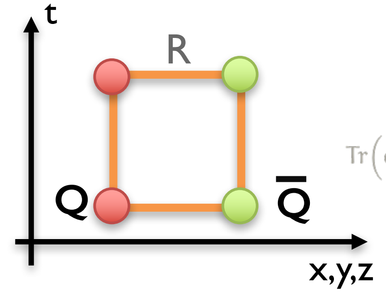

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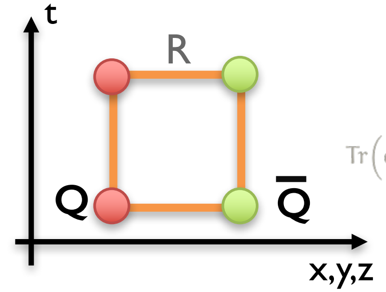

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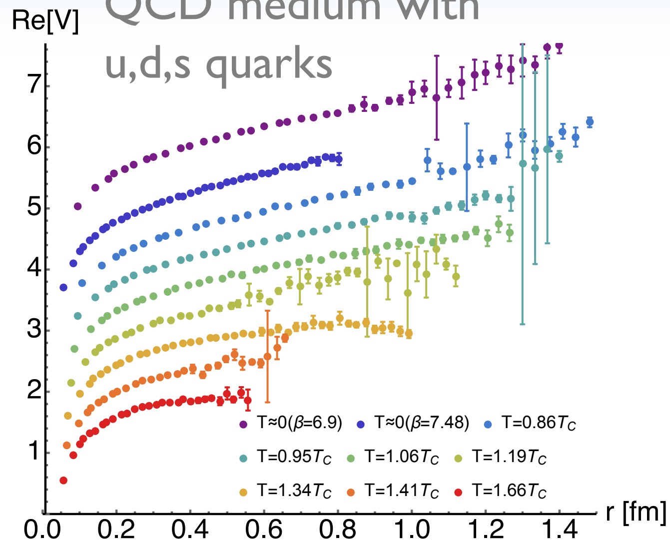
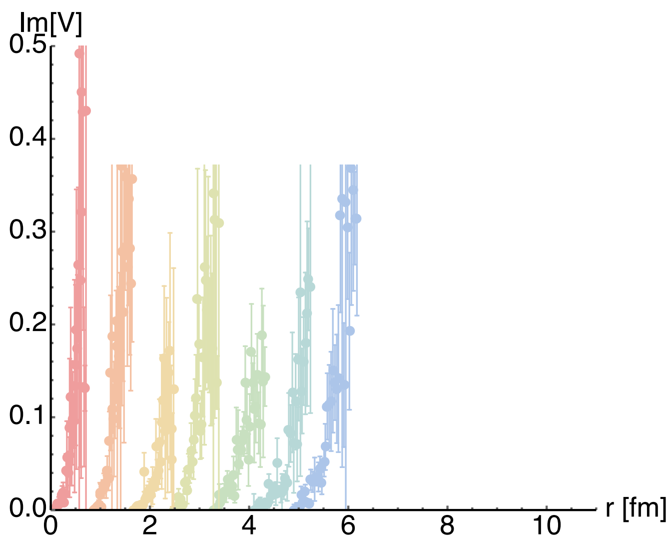
see A.R., T.Hatsuda & S.Sasaki , PRL 108 (2012) 162001, Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

T>0 static potential from the lattice

QCD medium with u,d,s quarks

Y.Burnier, O.Kaczmarek, A.R. PRL 114 (2015) 082001





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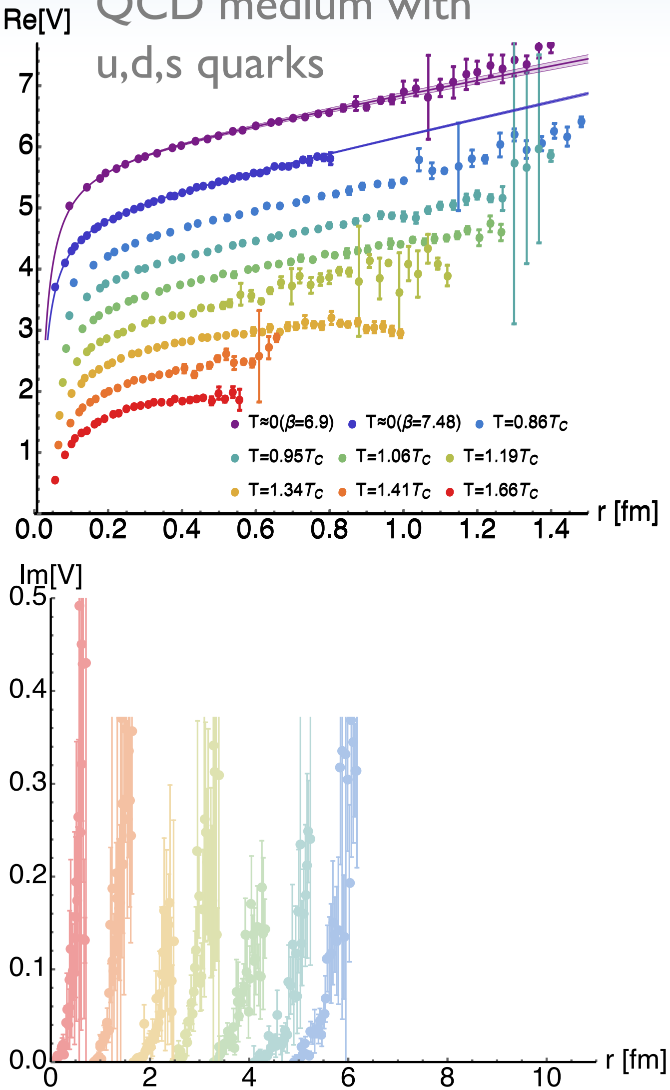

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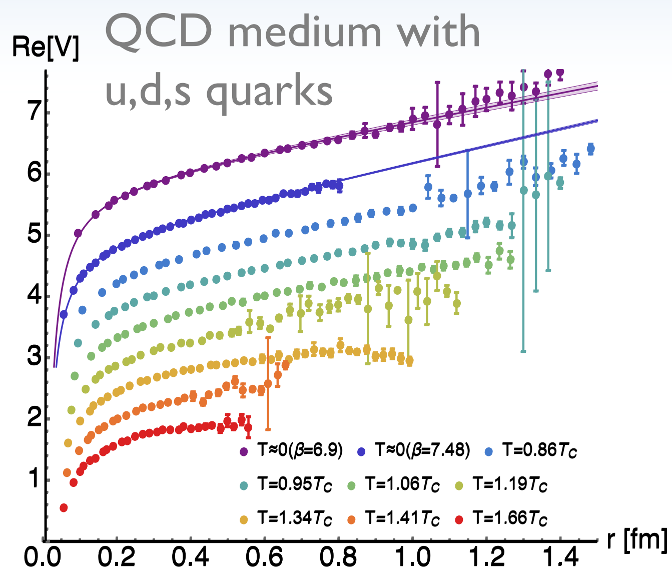

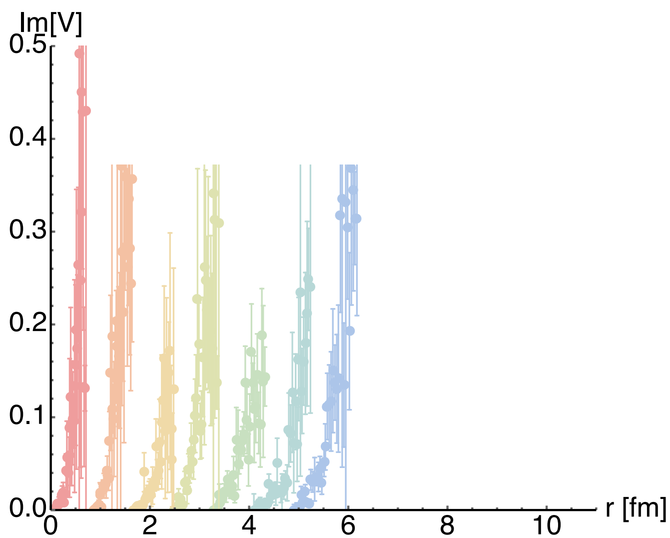

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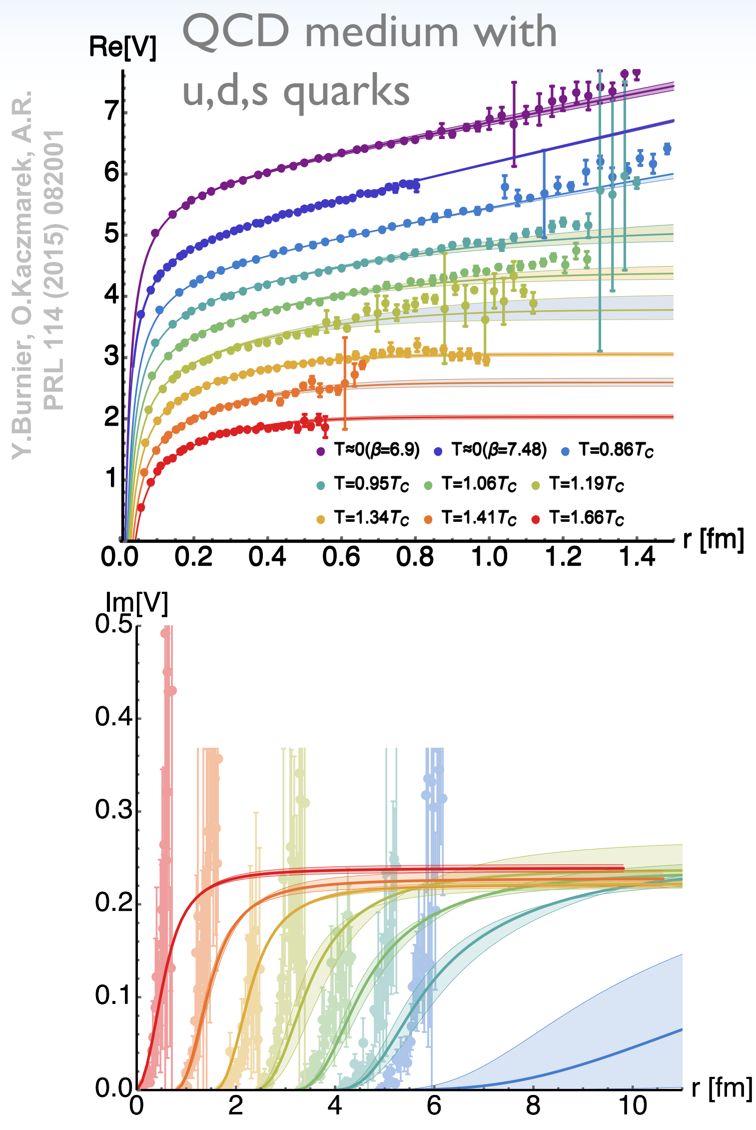
Y. Burnier, A.R. PLB753 (2016) 232

T>0 static potential from the lattice



Nf=2+1, 48^3x12, asqtad action, m_π~300MeV

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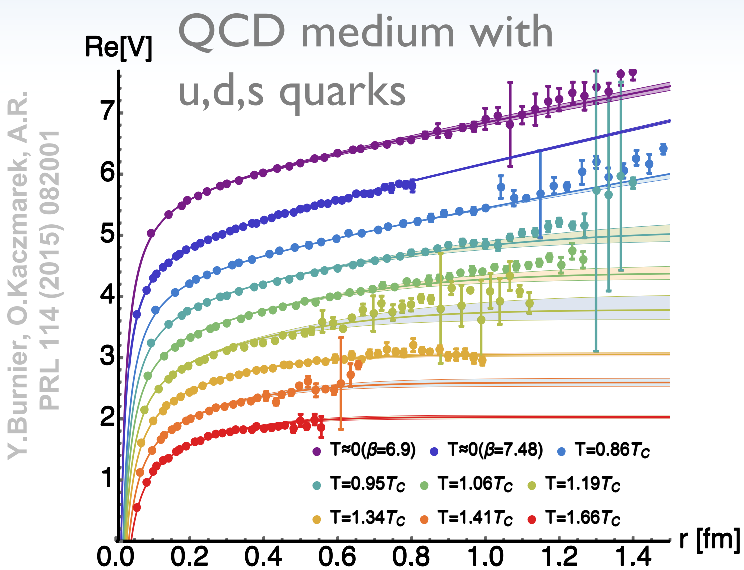

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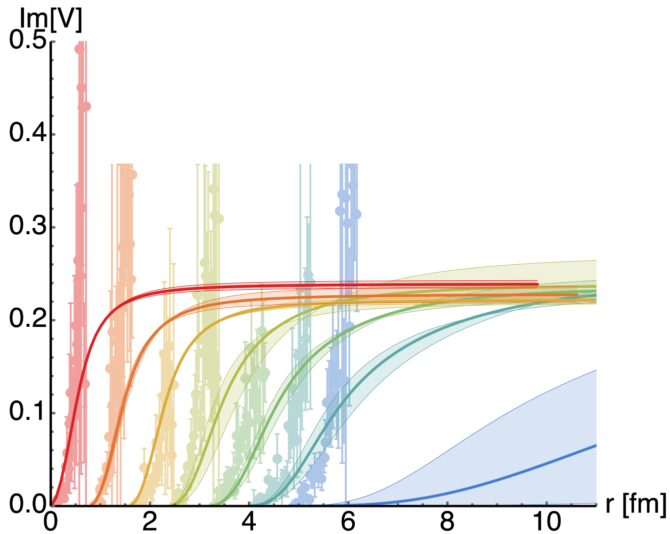

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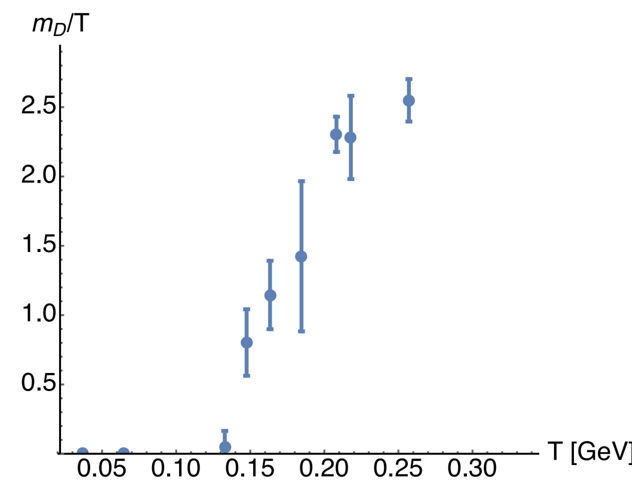

Y.Burnier, O.Kaczmarek, A.R.
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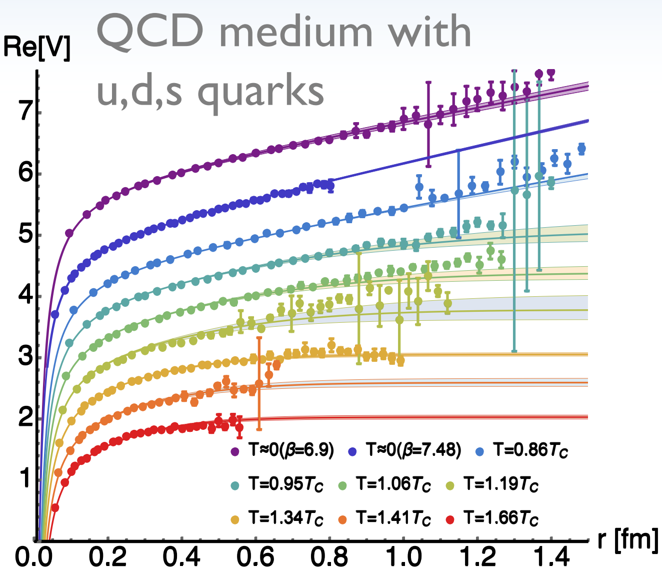
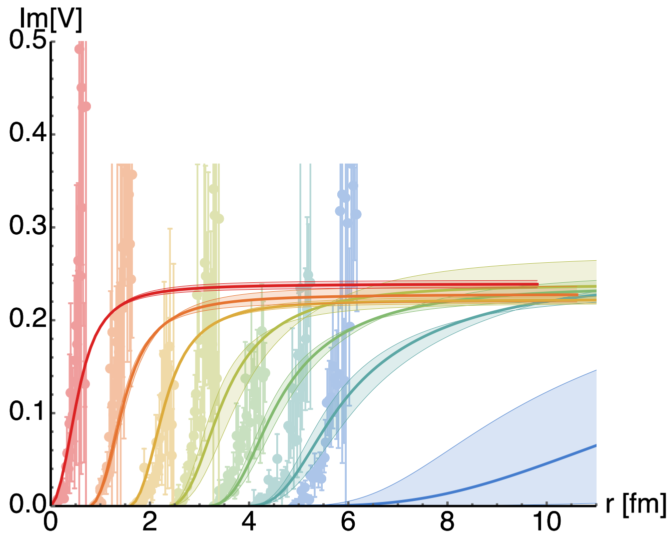
Y.Burnier, O.Kaczmarek, A.R. PRL 114 (2015) 082001





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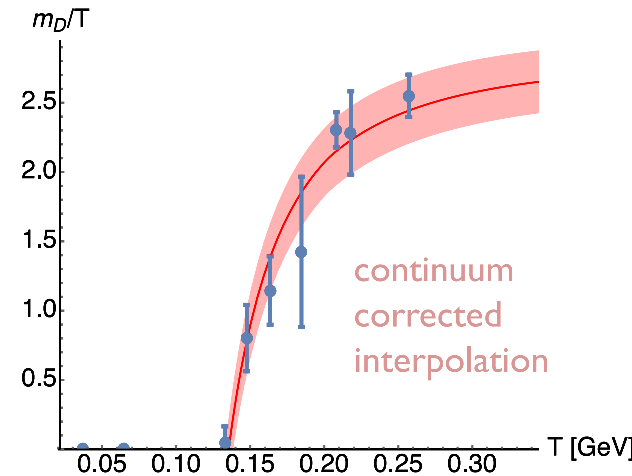

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S-wave spectral functions

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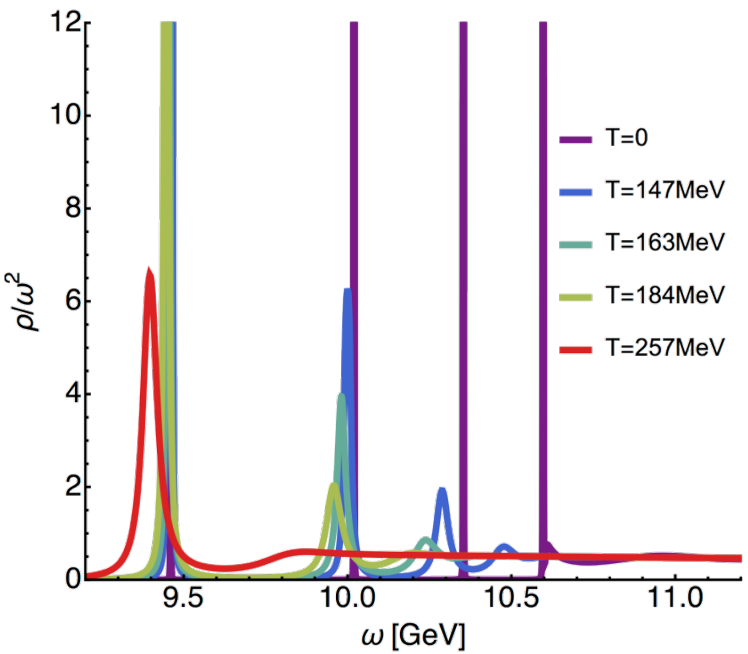
$^3S_1$ Bottomonium Υ channel



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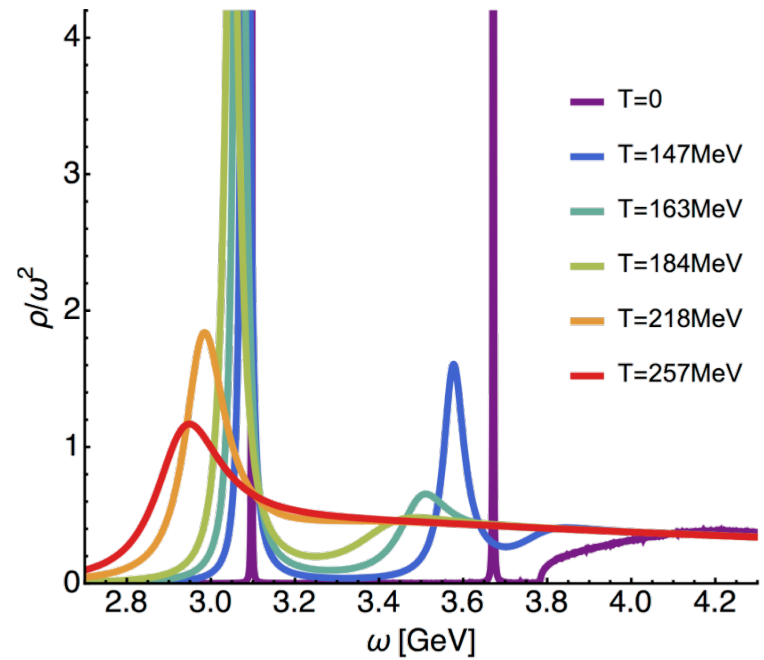

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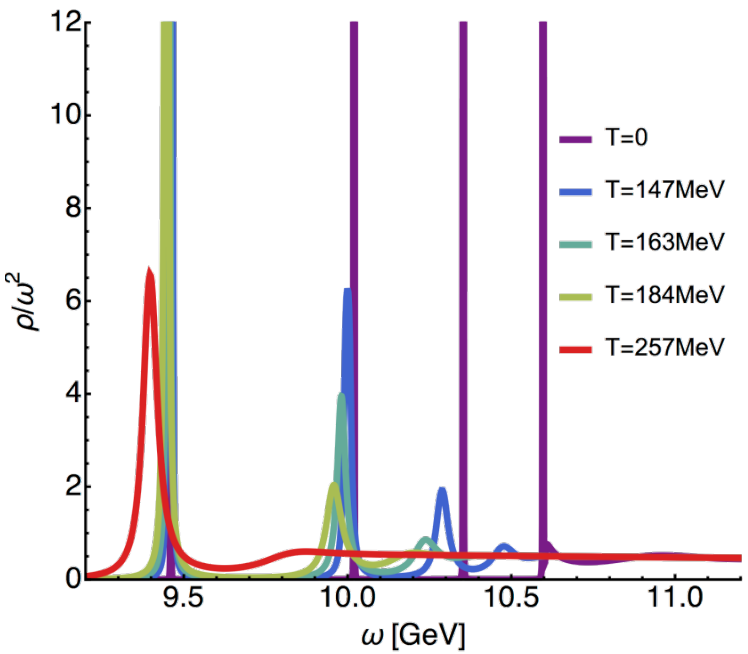

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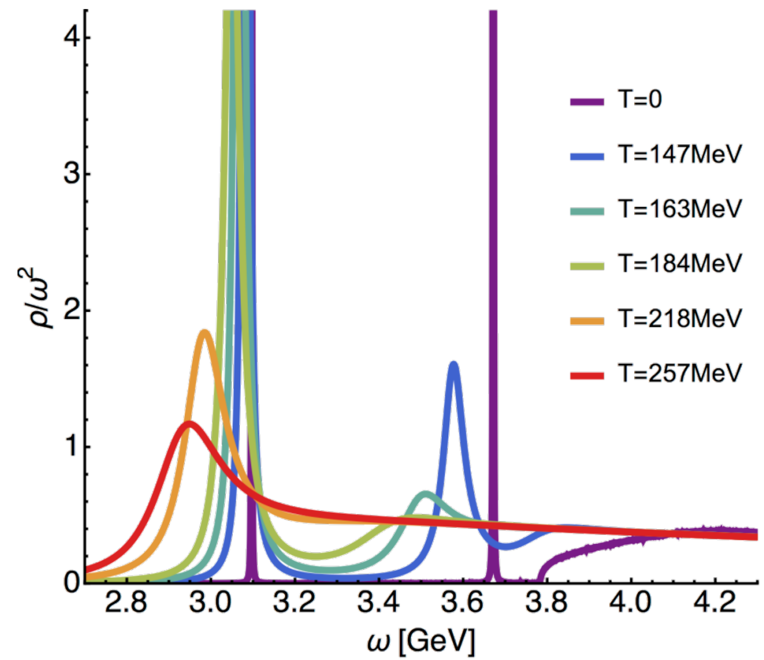
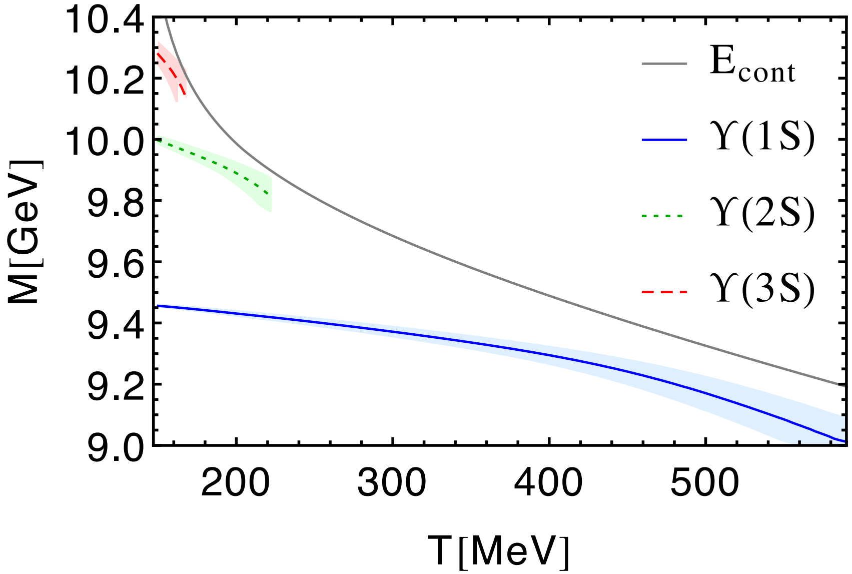
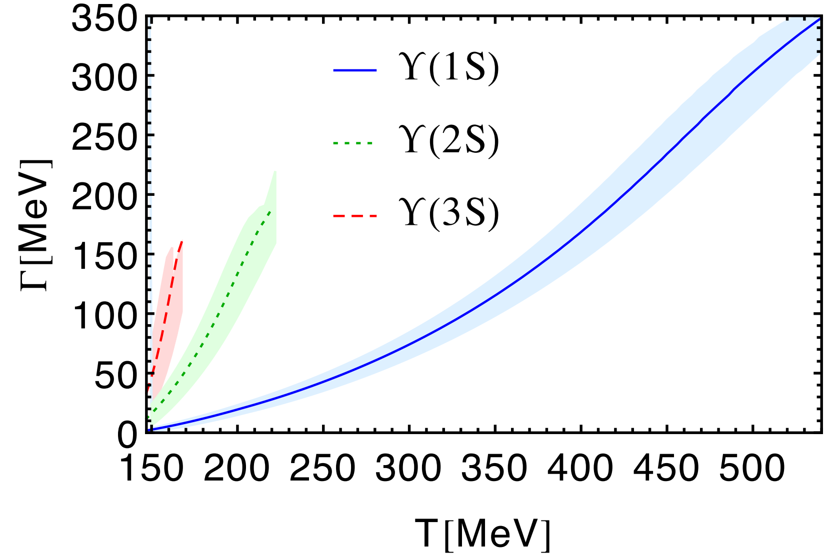


- Hierarchical modification of states according to their vacuum binding energy

Melting Temperatures: S-wave





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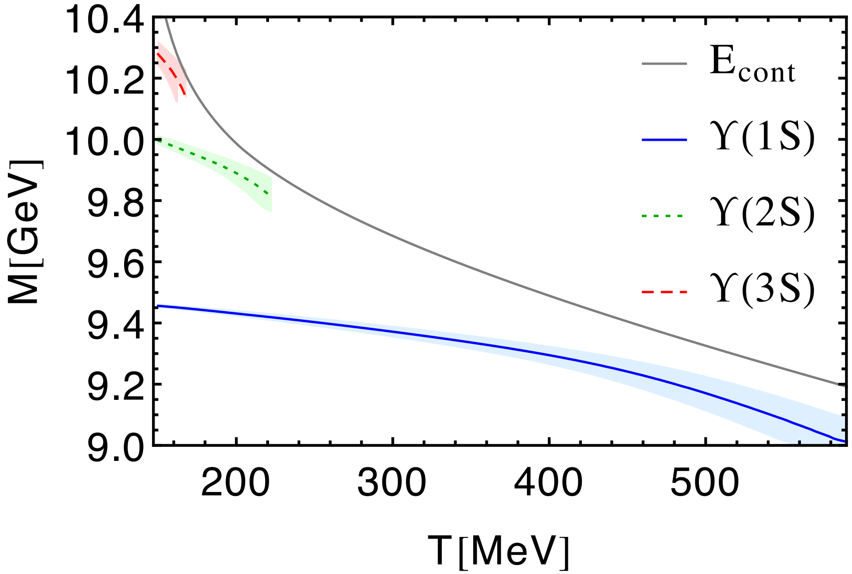

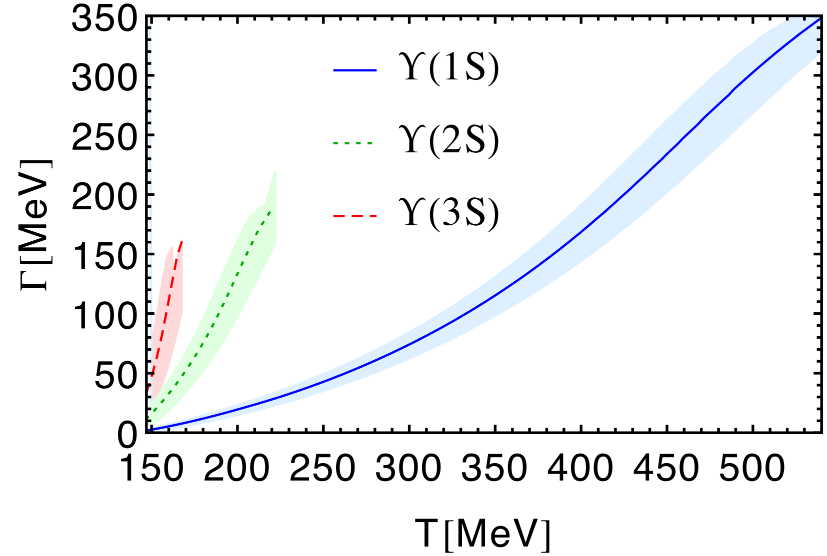

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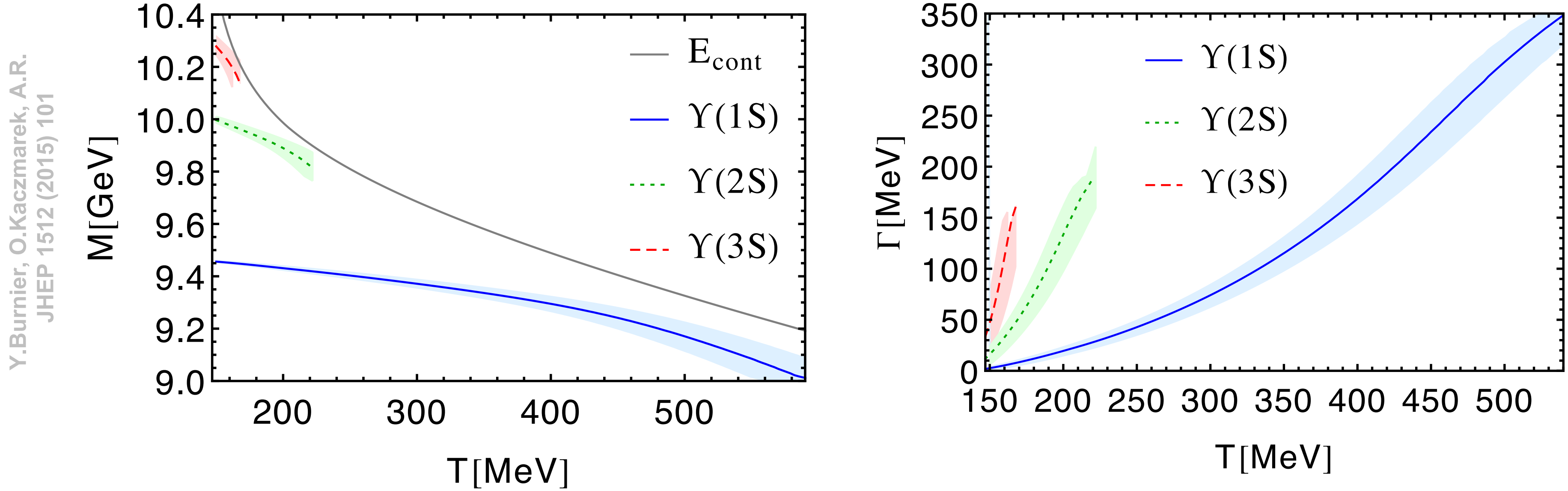

- In-medium modified Re[V]: threshold from confinement moves to lower energies
- Meaningful definition of melting in the presence of Im[V]: use $\Gamma = E_{bind}$

state	$J/\Psi(1S)$	$\Psi'(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$	
T_{melt}	213^{+13}_{-11}	< 147	412^{+76}_{-22}	193^{+26}_{-8}	157^{+5}_{-4}	< 147	[MeV]

Quarkonium phenomenology from in-medium spectral functions

Caveat I: We do not measure in-medium di-lepton emission in experiment instead the decay of vacuum states long after the QGP ceased to exist

Caveat II: The assumption of full kinetic thermalization is (if at all) only appropriate for Charmonium (at low p_T and at mid-rapidity y~0)

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→ Estimating the ψ' to J/ψ ratio

ψ' to J/ψ ratio from T>0 spectra

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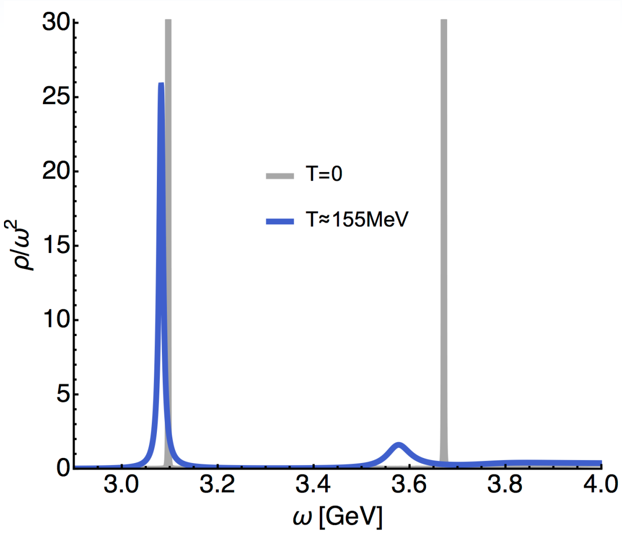

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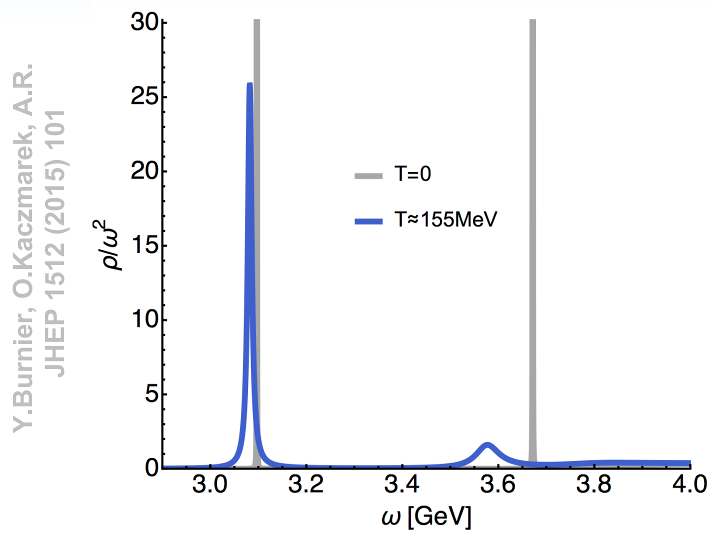

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$$R_{\ell\bar{\ell}} \propto \int dp_0 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\rho(P)}{P^2} n_B(p_0)$$

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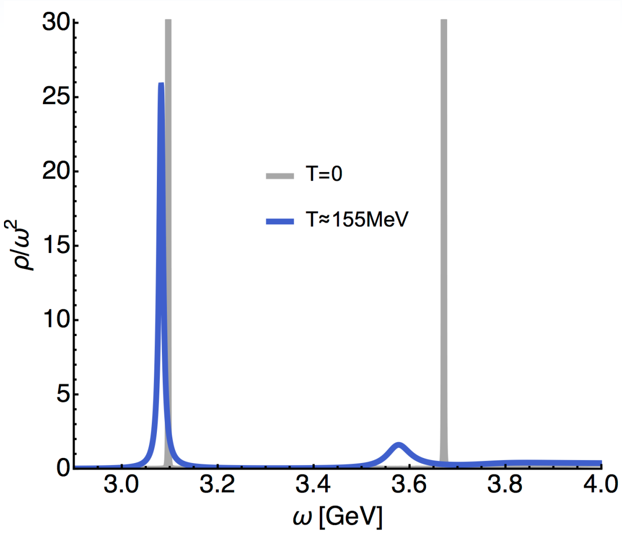

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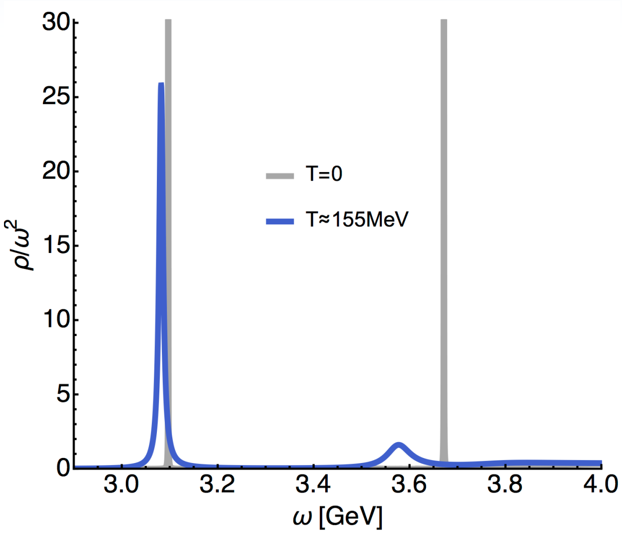

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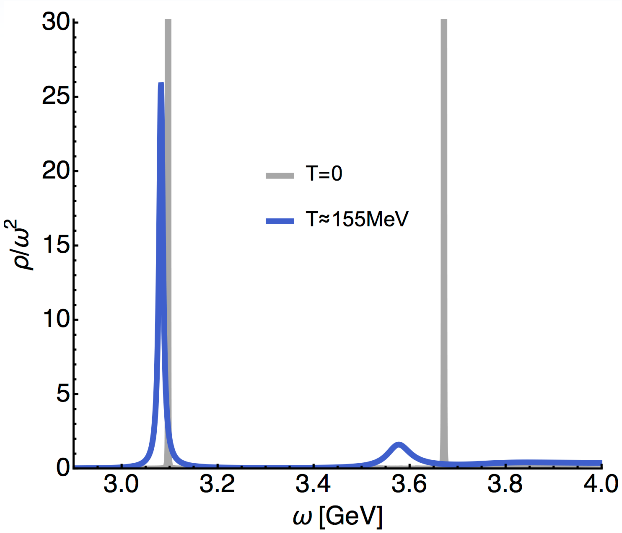

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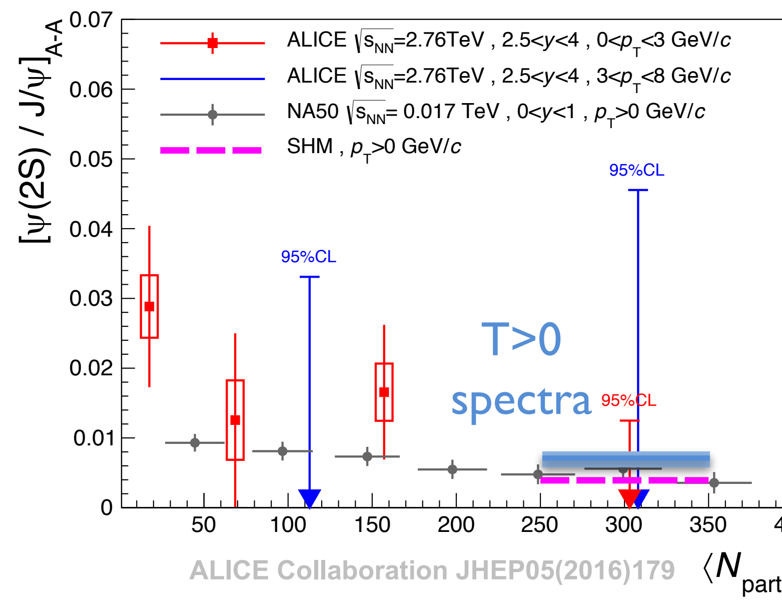

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    spectra challenging but show reasonable disappearance of bound state features
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- Significant improvement of spectral reconstructions on the horizon
  a new simulation prescription in imaginary frequencies (arXiv:1610.09531)

Thank you for your attention

Благодарю вас за внимание

Backup slides

Defining the T>0 $Q\bar{Q}$ potential

- Effective field theory $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$

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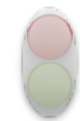
Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

Quantum mechanics



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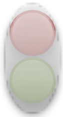
Relativistic thermal field theory

QCD

Dirac fields

$\bar{Q}(x), Q(x)$

Quantum mechanics



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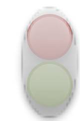

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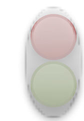

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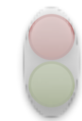
Singlet/Octet

$\psi_S(R,t), \psi_O(R,t)$



$\bar{q}(x), q(x), A^\mu(x)$

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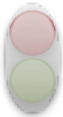

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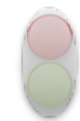

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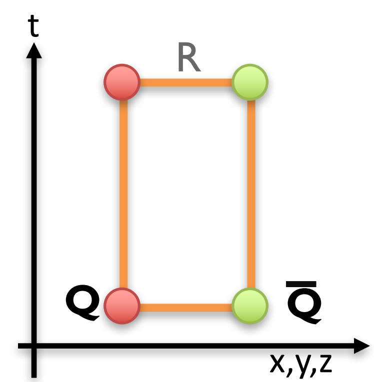
$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{pNRQCD} \equiv W_{\square}(R,t) = \mathrm{Tr}\left(\exp\left[-i\int_{\square} dx^{\mu} A_{\mu}(x)\right]\right)$$



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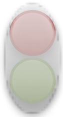
Quantum mechanics



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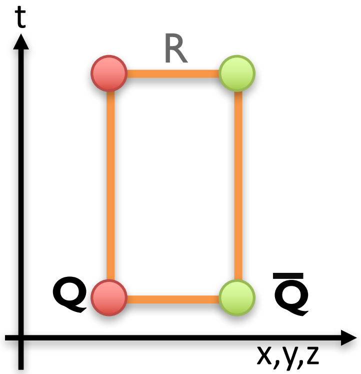

Defining the T>0 $Q\bar{Q}$ potential

- Effective field theory $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

QCD	NRQCD	pNRQCD
Dirac fields	Pauli fields	Singlet/Octet
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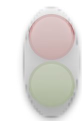

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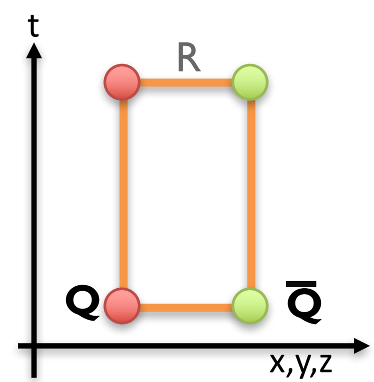

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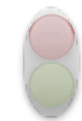

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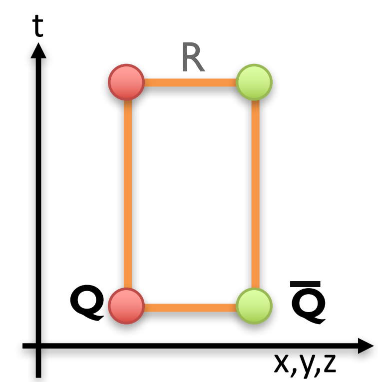

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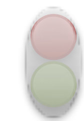

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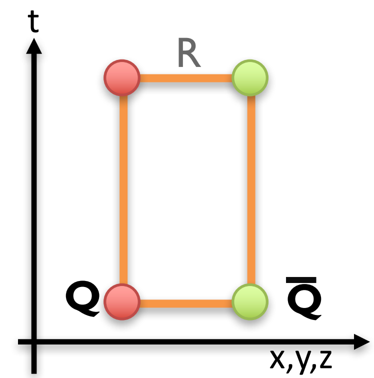

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$$\phi(x) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin[zx]}{zx}\right]$$

Laine et al. JHEP03 (2007) 054; Beraudo et. al. NPA 806:312,2008

Extracting V^{QCD} from lattice QCD

- On the lattice real-time observables not directly accessible!

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A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

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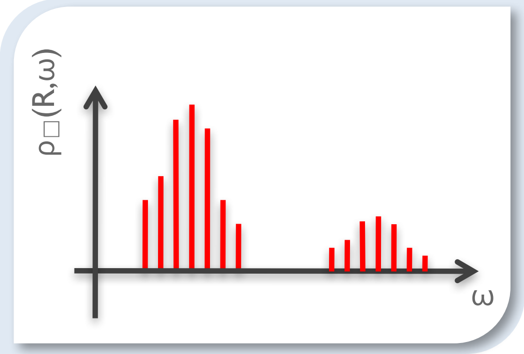

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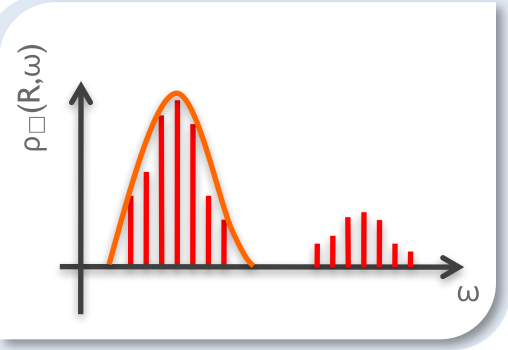

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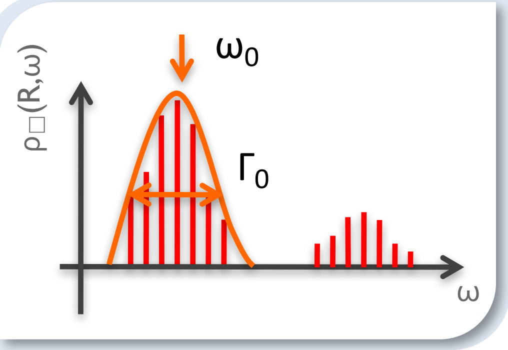

$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R)\cos[\gamma_2(R)] - (\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R)-\omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R)-\omega) + \dots$$

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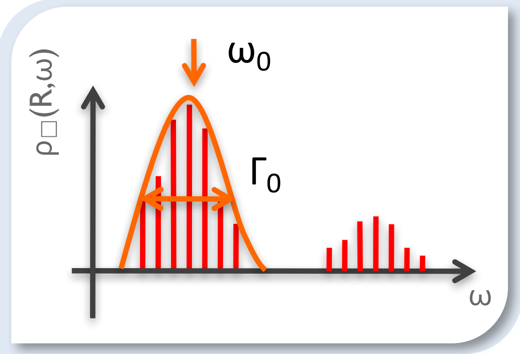

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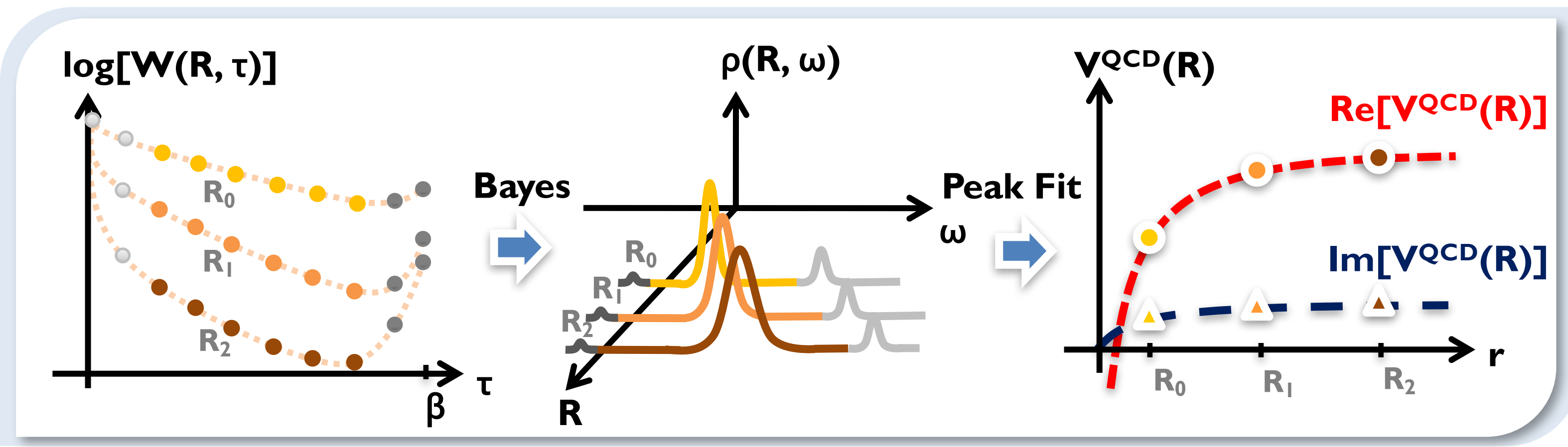
$$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$$

technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

Summary: V^{QCD} from the lattice

- From lattice QCD correlators to the complex heavy quark potential

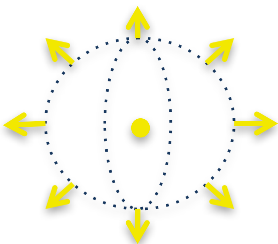


A.R. Mod. Phys. Lett. A, 28, 1330005 (2013)

- Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
  Practical reason: Absence of cusp divergences, hence less suppression along τ

The Gauss-Law Ansatz for V^{QCD}

- Towards phenomenology: Analytic expression for Re[V] and Im[V] necessary



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Strategy:

α_s,σ and c are vacuum prop. and do not change with T

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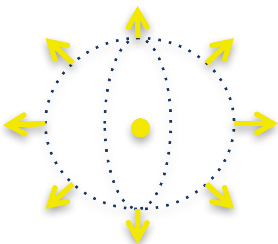

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V. V. Dixit,
Mod. Phys. Lett. A 5, 227 (1990)

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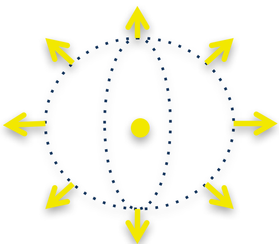

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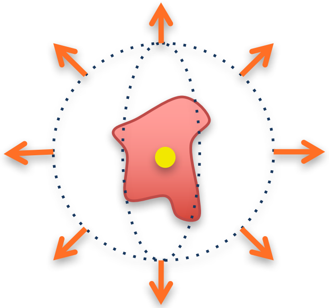

In the classical theory of Debye: Boltzmann distr. backgr. charges <ρ>

$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\, \alpha\left(\delta(\vec{r}) + \langle\rho(\vec{r})\rangle\right)$$

P. Debye, H. Hückel,
Phys.Z. 24, 185-206 (1923)

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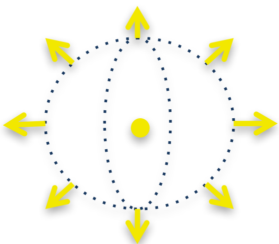

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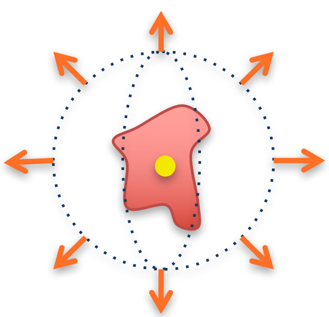

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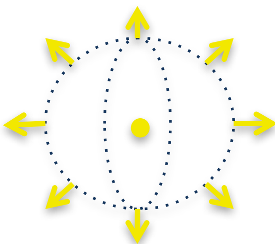

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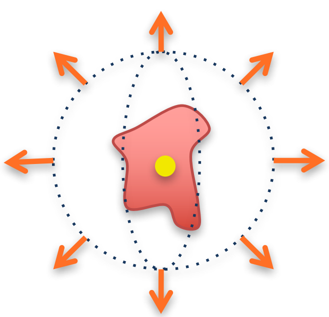

$$g(x) = 2\int_0^\infty dp\, \frac{\sin(px)}{px}\frac{p}{p^2+1}$$

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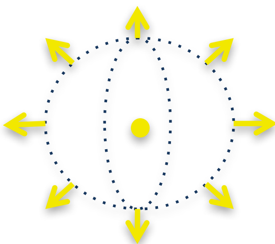

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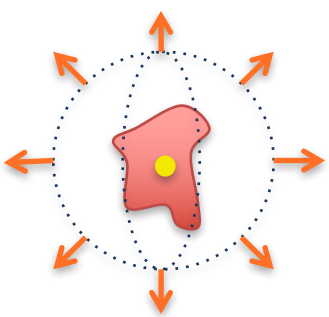

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$$g(x) = 2\int_0^\infty dp \frac{\sin(px)}{px}\frac{p}{p^2+1}$$

- Explicit solutions $Re[V]=Re[V_C]+Re[V_S]$ $Im[V]=Im[V_C]+Im[V_S]$ T-dependence only via $m_D(T)$

Heavy Quarks on the Lattice

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- Effective field theory from scale separation: $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$

Brambilla et. al.
Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

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Relativistic thermal field theory

QCD	NRQCD
Dirac fields	**Pauli fields**
$\bar{Q}(x), Q(x)$	$\chi^\dagger(x), \chi(x)$ $\xi^\dagger(x), \xi(x)$

$$L_{NRQCD} = \chi^\dagger\left(iD_t + \frac{D_i^2}{2M_Q} + \ldots\right)\chi + \xi^\dagger(\ldots)\xi$$

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- Individual Q or anti-Q in a medium background: Initial value problem $G(\tau) = \langle \chi(\tau)\chi^\dagger(0) \rangle$

$$G(\mathbf{x}, \tau + a) = U_4^\dagger(\mathbf{x}, \tau)(1 - \frac{\mathbf{p}_{lat}^2}{4M_Q a} + \ldots)G(\mathbf{x}, \tau)$$

well behaved if $M_Q a > 1.5$

Davies, Thacker Phys.Rev. D45 (1992)

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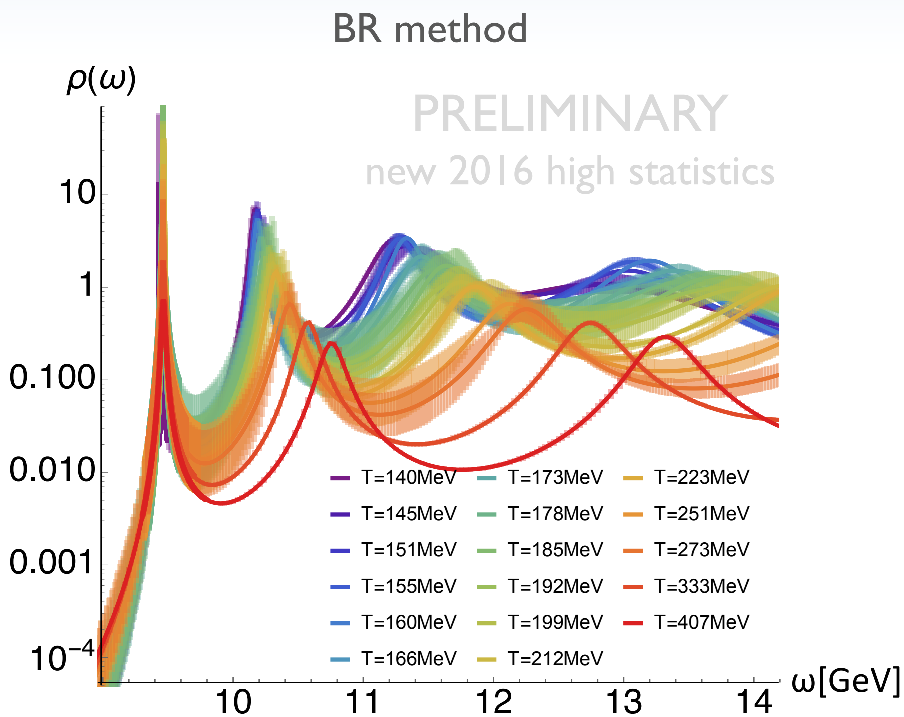
- $^3S_1$ (Υ) and $^3P_1$ (χ_{b1}) channel correlators $D(\tau)$ from products of heavy quark propagators $G(\tau)$

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau) G_{\mathbf{x}\tau} O^\dagger(\mathbf{x}_0, \tau_0) G^\dagger_{\mathbf{x}\tau}\rangle_{med} \qquad O(^3S_1; x, \tau) = \sigma_i, \quad O(^3P_1; x, \tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

Thacker, Lepage Phys.Rev. D43 (1991)

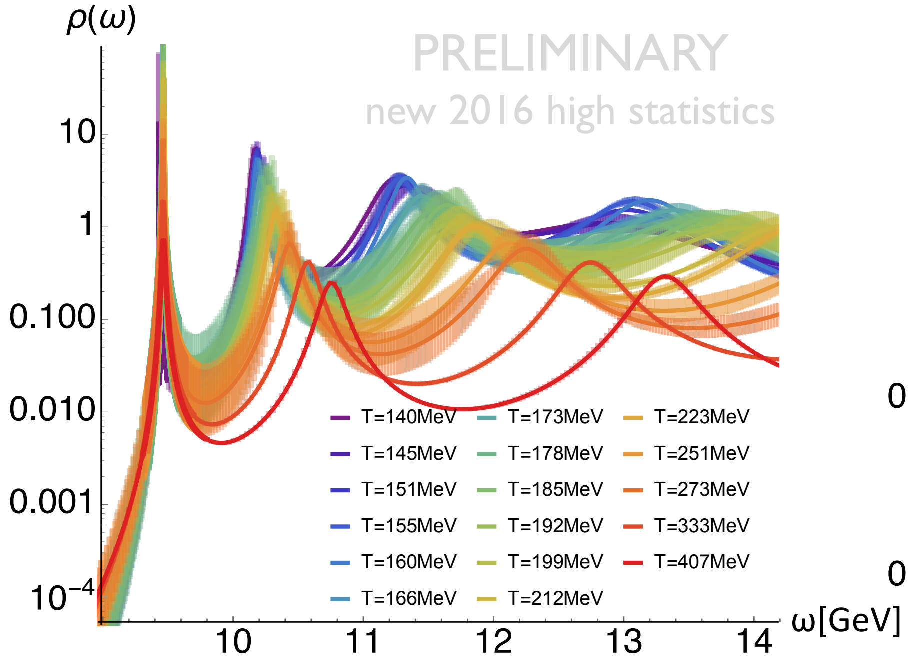
Bottomonium S-wave



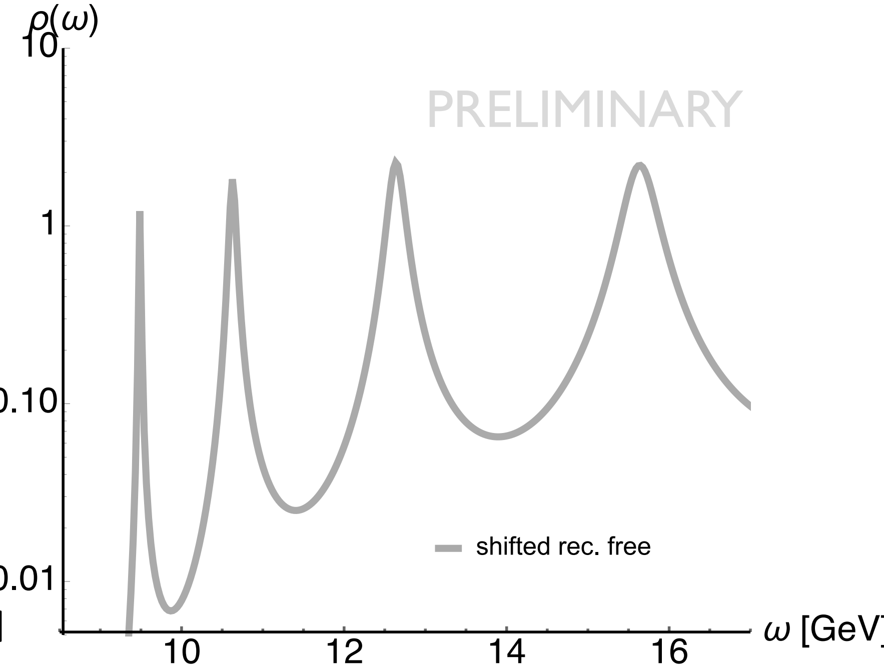
- Due to small N_τ=12: Bayesian reconstruction captures only ground state reliably
- BR method shows ground state feature at all temperatures T≤407MeV
- MEM: above T=333 MeV only washed out bump visible
- Systematics: MEM oversmoothing, BR ringing – use both methods to bracket

Bottomonium S-wave

BR method



BR reconstructed free vs. interacting spectrum



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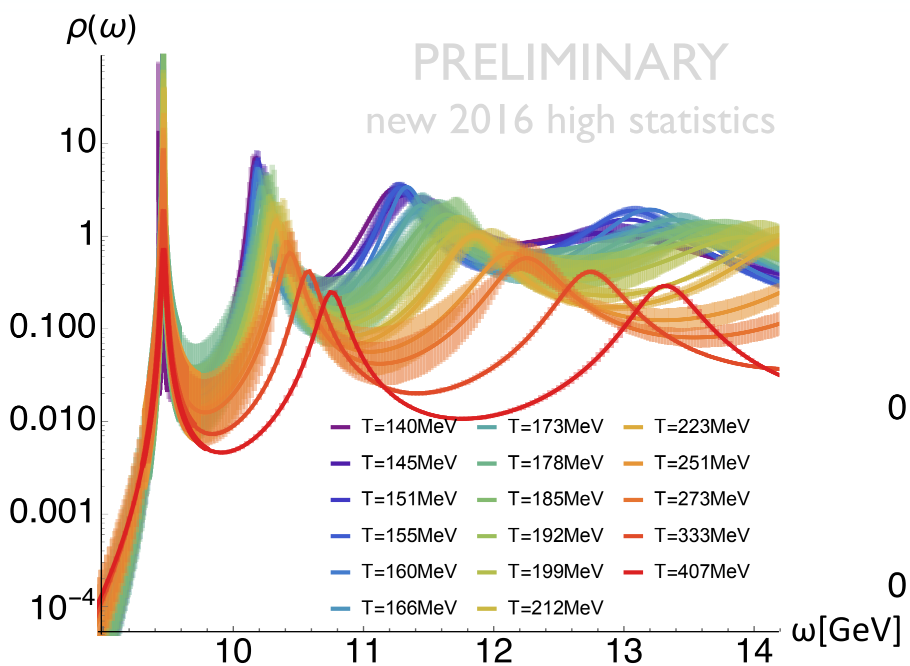

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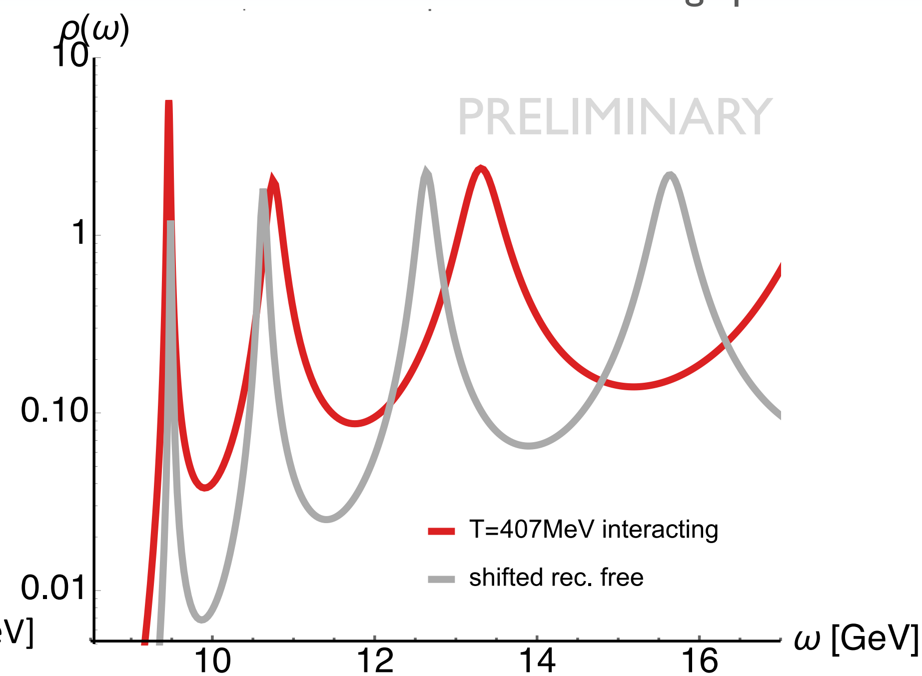

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